A proposal of a hydrodynamic model to low Reynolds numbers in a liquid-solid inverse fluidized bed reactor


This paper examines the fundamental characteristics of an inverse liquid-solid inverse fluidized bed reactor. In this reactor, many experiments were performed using three sizes of polypropylene spherical solids, lighter than water, with different diameters (4.00, 4.16 and 4.18 mm) and of densities ranging from 808 to 867 kg/m³, these were fluidized by continuous liquid phase flow. Various bed heights were obtained at different flowrates from 0.95 to 9.5 (L min⁻¹).

A predictive model is developed for the hydrodynamic behavior, such as height and solid holdups (bed porosity), this model was derived from the balance of forces acting on a single particle and the set of particles found in the fluidized bed; uses process parameters, such as reactor dimensions, particle properties, and liquid flowrates as input variables, this model also includes the inertial drag coefficient which allow extended a non-spherical particles. This model has several advantages when compared with previously reported. Among them, advantages such as standard deviation values ≤0.9% between experimental and calculated bed porosity, the bed porosity can be expressed in terms of the Reynolds and Archimedes numbers and this model estimate bed porosity in a range of Reynolds number from 5.5 to 200 which has not been studied in an inverse fluidized bed reactors.

**Keywords**: Inverse fluidized bed reactor, bed porosity, porosity, reynolds number, archimedes number, hydrodynamic, solid holdups, inertial drag coefficients.

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El presente estudio tuvo como finalidad estudiar el comportamiento hidrodinámico de un reactor de lecho fluidizado sólido-líquido operado a flujo inverso. En este reactor se realizaron varios experimentos usando partículas esféricas sólidas de polipropileno más ligeras que el agua de tres diferentes tamaños, con distintos diámetros (4.00, 4.16 y 4.18 mm) y densidades que van de 808 a 867 kg/m³, que fueron fluidizadas por un flujo continuo de líquido. La altura del lecho fue determinada a diferentes velocidades de flujo de 0.95 a 9.5 (l min⁻¹). Se desarrolló un modelo para predecir el comportamiento hidrodinámico, como la altura y el vacío entre los sólidos (porosidad del lecho); este modelo se derivó de la relación de fuerzas que actúan sobre una sola partícula y el conjunto de partículas que se encuentran en el lecho fluidizado; utiliza parámetros del proceso, como dimensiones del reactor, propiedades de las partículas y velocidades del líquido como variables de entrada; este modelo también incluye el coeficiente de arrastre inercial, el cual permite aplicarlo para partículas no esféricas. Este modelo tiene varias ventajas comparado con los reportados por otros autores, entre ellas, valores de desviación estándar ≤ 0.9% entre la porosidad del lecho experimental y la calculada; la porosidad del lecho puede ser expresada en términos de los números de Reynolds y Arquímedes, y este modelo estima la porosidad del lecho en un rango de número de Reynolds a partir de 5.5 a 200, el cual no ha sido estudiado en un reactor de lecho fluidizado inverso.

**Palabras clave**: reactor de lecho fluidizado inverso, porosidad del lecho, porosidad, número de Reynolds, número de Arquímedes, hidrodinámica, vacío entre sólidos, coeficientes de arrastre inercial.
Introduction

Fluidization is an operation in which solid particles enters in contact with a fluid acquiring fluid characteristics (Mukherjee, Mishra, & Ran, 2009) and is known for its wide use since the early 50’s. In conventional fluidization, the solid particles have a higher density than the fluid therefore the bed solids can be fluidized by an upwards flow, in the case of a bed of particles having a density smaller than the fluid (usually liquid), the bed is fluidized by a downwards flow of the liquid and it is usually called inverse fluidized bed (Bimal, Uma, & Sudip, 2010). Among the advantages presented by the latest are high mass transfer rates, minimum carry over of coated microorganisms due to less solid attrition, efficient control of biofilm thickness and ease of refluidization in case of power failure (Garcia, Buffière, & Elmaleh, & Moletta, 1998; Bimal et al., 2010). These significant advantages found many applications of inverse fluidized beds in biochemical processes like ferrous iron oxidation and aerobic and anaerobic biological wastewater treatment like treatment of wine distillery wastewater. Previous studies on liquid-solid inverse fluidized bed reactor (IFBR) dealt with different hydrodynamic characteristics such as pressure drop, minimum fluidization velocity and bed void fraction e.g. (Fan, Muroyama, & Chern, 1982; Karamanev & Nivkolov, 1992; Renganathan & Krishnaiah, 2007).

The bed void fraction is one of the important design parameters that determine the height of fluidized bed during steady state operation. Under normal operating conditions, fluidized bed reactors operate under steady state. However, unsteady state operation is encountered during start-up and shut-down of the reactor and due to fluctuations in liquid flow rates. Such conditions may prevail in a wastewater effluent treatment plant where an IFBR is mostly used. Therefore, information on the unsteady state bed expansion is required for better understanding of the fluid dynamics under transient conditions, which will lead to better control of the reactor and for the design of IFBR an industrial scale (Renganathan & Krishnaiah, 2007).

In many processes there are particles with diameter different in the particles of the bed greatly modifies its porosity due to the particle’s size and density variation. This modification in the bed porosity affects the proper operation and control of the fluid bed reactor. Therefore, it is important to have a reliable model in order to estimate bed porosity for design and scale-up of fluid bed reactors.

Numerous models have been proposed to predict the bed porosity in fluidized beds with spherical and non-spherical particles, but only it includes a range of Reynolds number from 200 to 6,000. The major and most used until today by several authors (Yang & Renken, 2003; Akgiray & Soyer, 2006; Renganathan & Krishnaiah, 2007; Fuentes, Scena, Aguirre, & Mussati, 2008; Soyer & Akgiray, 2009) are: Richardson and Zaki (1954), Wen and Yu (1966), Ramamurthy and Subbaraju (1973), Riba and Couderc (1977), Fan et al. (1982), and Setiadi (1995) models.

Mathematical models for prediction of traditional fluidized bed porosity can be used in inverse fluidization, nevertheless, some authors disagrees with this practice (Wen & Yu, 1966; Ramamurthy & Subbaraju, 1973; Hyun, 2001).

Therefore the general aim of this work is to study the hydrodynamic of a liquid-solid inverse fluidized bed reactor with different polypropylene spherical particles diameters and different densities to propose a mathematic model to estimate bed porosity in a range of Reynolds number from 5.5 to 200, which has not been studied in this reactor and this hydrodynamic characteristics are very important to low fluidization rates in biotechnology process.

Materials and methods

Liquid and solid phase

Water at 20 °C was used as fluid work in an inverse fluidization reactor and polypropylene spherical particles of average diameter 4.00, 4.16 and 4.18 mm and density of 808, 825 and 867
kg/m³ were used as solid phase. The sphericity is an important measure that describes how round a particle is defined as the ratio of the surface area of a sphere with the same volume as the given particle to the surface area of the particle itself, therefore the solid particles in this work have a sphericity of 1.

**Installation**

A schematic diagram of the fluidized bed reactor used in this work is shown in Figure 1. As shown, the experimental setup included a 0.04 m of internal diameter (ID) and 0.50 m high cylindrical glass column. The work fluid was stored in a tank, and pumped into the column by a 1/8 HP little giant centrifugal pump, model 583002. The flow rate in the reactor was measured with a rotameter and fixed in the range between 0.95-9.5 (l min⁻¹).

**Start-up**

The inverse fluidization with each particles lot was done of the following way: The inverse fluidized bed reactor was filled with water at 20 °C, the water was used as a liquid phase, the polypropylene particles were introduced for the upper part of the reactor and the initial height bed was measured with the calibrated scale in centimeter. The flow rate was increased 6 times more (0.63, 0.88, 1.00, 1.13, 1.26 and 1.38 l min⁻¹) and the heights bed was measured. Figure 2 shows the inverse fluidization reactor at different flowrates.

**Particle density**

Particle density was determined by Standard test methods for specific gravity of solids by water pycnometer (ASTM D854-14) to determining their volume and mass, this method was repeat three times. The volume was measured using a pycnometer filled with light oil (density 750 kg/m³) and the mass was measured in an analytical balance Sartorius ±1x10⁻⁷ kg. The particle density was calculated by dividing the mass by the volume.

The density media was determined by pycnometer (ASTM D854-14). The viscosity media was measured with a Malvern SV-10 vibratory viscosimeter.

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Figure 1. Schematic diagram of the inverse fluidized bed reactor.
Results and discussion

Hydrodynamic study

The different equations that relate the bed porosity with the fluid superficial rate can be classified into three main groups Fan et al. (1982). The first group is based on a relationship that dependence between velocity and initial velocity (U/Ui) and porosity (e). The Richardson and Zaki (1954) equation is the most popular of this group. The second group of equations is based on the drag for multi-particulates systems. This is usually given as a function of Reynolds number (Re) and Archimedes number (Ar). Ramamurthy and Subbaraju (1973), and Riba and Couderc (1977) are typical for this group.

The third group is based on the ratio obtained from the balance of forces acting on a particle isolated (gravity, floating, drag) and assume that applies to all particles in the bed as the equation of Wen and Yu (1966) for spherical and non-spherical particles using a shape factor (ψ) and Reynolds numbers are in the range 200 to 6000. Table 1 shows the function of porosity shown Wen and Yu (1966), Riba and Couderc (1977), and Ramamurthy and Subbaraju (1973) which were chosen because they have a lower standard deviation in the case of spherical particles of uniform size.

Predictive model proposed and comparison with other models

The comparison of the results was made by plotting the porosity function obtained from values proposed by Wen and Yu (1966), Ramamurthy and Subbaraju (1973), Riba and Courdec (1977) and this work, versus experimental bed porosity data.
Table 1. Porosity function model by different authors.

<table>
<thead>
<tr>
<th>Author</th>
<th>Porosity function</th>
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<tbody>
<tr>
<td>Wen and Yu (1966)</td>
<td>$f(\varepsilon)_{WY} = \frac{Ar}{18Re+2.7Re^{1.687}}$</td>
</tr>
<tr>
<td>Riba and Courdec (1977)</td>
<td>$f(\varepsilon)_{RC} = \frac{GaMv}{1.9(18Re+2.7Re^{1.687})}$</td>
</tr>
<tr>
<td>Ramamurthy and Subbaraju (1973)</td>
<td>$f(\varepsilon)_{RS} = \left(\frac{Ar}{13.9(Re^{1.3})}\right)^{1/3}$</td>
</tr>
</tbody>
</table>

Ar: Archimedes number $= \frac{d_p^3 g \rho_l (\rho_s - \rho_p)}{\mu_r^2}$

$\rho_p$: Particle diameter

$f(\varepsilon)$: Porosity function

$g$: Gravity

Ga: Galileo number $= \frac{d_p^3 g \rho_l^2}{\mu_r^3}$

$Mv$: Density ratio $= \frac{\rho_s - \rho_p}{\rho_p}$

$\mu_r$: Liquid viscosity

Re: Reynolds number $= \frac{d_p v L}{\mu_r}$

$\rho_l$: Liquid density

$\rho_s$: Particle density

The model proposed in this paper was designed using the inertial drag coefficient ($C_i$) Becker (1959) for $Re < 200$. The data used in the model proposed in this work were Wilhelm and Kwauk (1948) and the experimental data obtained in this work. The model to calculate the porosity of spherical particles and the application intervals as shown in table 2.

Ramamurthy and Subbaraju (1973) porosity function fits the experimental data tendency, but do not represent the experimental bed porosity. The plot (figure 3) shows an experimental underestimation for experimental data between 0.5 and 0.73 of bed porosity, this deviation can be attributed to the principal assumption by these authors that the solid particles are represented as an ensemble moving about points considered as nodes of an imaginary lattice through whose free volume the fluid is flowing. It is important to notice that this model was developed for annular fluid.

Riba and Courdec (1977) model fits the experimental data tendency but the plot shows a slight experimental underestimation for experimental data between 0.55 and 0.63 of bed porosity.

Wen and Yu (1966) model fits has a good correlation of bed porosity data; the standard deviation was 1.2%.

The mathematical model proposed in this work fits well the trend of the experimental bed porosity data for spherical particles which was well represented by this function in the interval of porosity from 0.47 to 0.73; the standard deviation was 0.9%.

The new model has several advantages because Wen and Yu (1966), Riba and Courdec (1977), and Ramamurthy and Subbaraju (1973), restrict their model to traditional fluidization (upward flow) and spherical particles. Furthermore the new model is considerably simple in form, it can be used for spherical particles, but includes a term of inertial drag coefficient proposed by Becker (1959) which includes a shape factor to non-spherical particles.

The figure 3 shows the graph of the function of porosity that results from using the correlation Wen and Yu, Riba and Courdec (1977), Ramamurthy and Subbaraju (1973), and the model proposed in this work.
Table 2. Model proposed in this work to calculate the porosity of spherical particles.

<table>
<thead>
<tr>
<th>Model proposed in this paper</th>
<th>Application intervals</th>
</tr>
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</table>
| $1.8 e^{-2.74} = \frac{A_r}{0.75(24 Re_s + C_i Re_s)}$ | $5.5 \leq Re_s \leq 200$
|                              | $0.47 \leq \varepsilon \leq 0.73$
|                              | Spheres               |

$A_r$ Archimedes number $\left[ = \frac{d_s^3 \rho_L (\rho_s - \rho_L)}{\mu_L} \right]$

$C_i$ Inertial coefficient

$d_s$ Solid diameter

$\varepsilon$ Porosity

$g$ Gravity

$\mu_L$ Liquid viscosity

$Re_s$ Solid Reynolds number $\left[ = \frac{d_s v_L}{\mu_L} \right]$

$\rho_L$ Liquid density

$\rho_s$ Solid density

The table 3 shows a comparison of the standard deviation of the equation proposed in this work and the equations published in literature.

Conclusions

The experiments were conducted in an inverse fluidized bed reactor at different flowrates. The results show that the approach to the balance of forces acting on a particle, namely gravity, the buoyancy force and the drag applied to a normal flow, i.e., from the bottom up, is valid for case of a reverse flow (from top to bottom). The equation Wen and Yu (1966) and the equation proposed in this work are showing a lower standard deviation to the experimental data.
The equation proposed in this paper can make a better approach by the balance of forces with a larger number of data in the literature. It is important to identify the study variables, also, to a dimensional analysis, since grouping variables in dimensionless numbers such as the Reynolds and Archimedes allow the handling of mathematical models takes just a simple manner, also, dimensionless numbers handling facilitates the design and scale reactors.

Acknowledgments

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References


<table>
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<tr>
<th>Equation</th>
<th>Standard desviation (%)</th>
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</thead>
<tbody>
<tr>
<td>Wen and Yu (1966)</td>
<td>1.2</td>
</tr>
<tr>
<td>Riba and Courderc (1977)</td>
<td>2.69</td>
</tr>
<tr>
<td>Ramammurthy and Subaraju (1973)</td>
<td>3.46</td>
</tr>
<tr>
<td>Model proposed in this work (2016)</td>
<td>0.9</td>
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Campos-Díaz & Alvarez-Cruz, A proposal of a hydrodynamic model to low Reynolds numbers in a liquid-solid inverse fluidized bed reactor

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