Composite Roughness in Hydraulic Models

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Abstract

Marengo-Mogollón, H., Aldama-Rodríguez, A., & Romero-Castro, I. Composite Roughness in Hydraulic Models. *Water Technology and Sciences* (in Spanish), 7(6), 159-178.

An experimental survey study was carried out on hydraulic models in tunnels working as full pipe on composite roughness; the experimental analysis was made with different friction coefficients (that belong to different materials) and compared with values calculated from a proposed theoretical formulation. The model was made for the diversion work of El Cajón hydroelectric project, these criteria also was applied to the Grijalva Tunnels under operation in Mexico and they are also being used for La Yesca hydroelectric project presently under construction.

Keywords: Acrylic-plastic-carpet, coefficient of resistance, composite roughness, hydraulic models.

Resumen

Marengo-Mogollón, H., Aldama-Rodríguez, A., & Romero-Castro, I. (noviembre-diciembre, 2016). Rugosidad compuesta en modelos hidráulicos. Tecnología y Ciencias del Agua, 7*(6), 159-178.*

En un aparato experimental se llevó a cabo un modelo en túneles hidráulicos trabajando a tubo lleno con rugosidad compuesta; el análisis experimental fue con distintos coeficientes de fricción (que pertenecen a distintos materiales). Se comparan con valores calculados a partir de formulaciones teóricas. El modelo se hizo para la obra de desvío del proyecto hidroeléctrico El Cajón, y se aplicó también para el diseño de los túneles de emergencia del Grijalva, ambos proyectos en operación en México.

***Palabras clave**: acrílico-plástico, coeficiente resistencia, rugosidad compuesta, modelos hidráulicos.*

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Introduction

With an analysis by overtopping of the diversion works of Aguamilpa Dam, in Mexico, on January 1992, Marengo-Mogollón (2006) concludes that: "*the overtopping event would have been avoided if a hydraulic concrete lining would have been built at the floor, and shotcrete at the walls and vault of the tunnels with vault type (16 × 16 m) section (composite roughness concept) even though the peak inflow rate exceeded by 50% the original design value*". This paper shows the analysis made in a hydraulic model with composite roughness that simulates permanent flow in tunnels working as full pipe. While the flow in prototype is mainly governed by gravity, in the model it is also influenced by viscosity nevertheless, working within a limited range of variation of the Reynolds number, and adopting Froude's similitude, has made possible to ignore such influence in all the analyzed models. The main objective of this paper is to validate the theoretical analysis made by Elfman (1993), Marengo-Mogollón (2005) and to prove the best criteria within five formulas that permit the estimation of the roughness coefficient. This paper is organized as follows: first, a brief description of the experimental apparatus is made, then is showed a brief hydraulics review of flow resistance equations, the hydraulic theoretical development in order to evaluate the composite roughness is presented, and the paper concludes by comparing the hydraulic experimental results of the Colebrook equation with the 13 criteria in the transition zone.

Experimental Apparatus

The experimental apparatus is shown in figure 1. The test section is 0,133 × 0,133 m, and has variable slope with a 9m length. The inlet geometries tested in the model and built in prototype, are rounded. The testes were made with each material acrylic, sandpaper, plastic and carpet (figure 2), in order to know the main hydraulic properties in each one of them. It was then tested a tunnel of compound roughness that was obtained when it was used the acrylic in the bottom, sandpaper, plastic and carpet in the walls and vault respectively in each one of them. In the analysis, it was considered that Froude's similitude law is applicable, considering that, the friction factor is independent of Reynolds' number.

Figure 1. Experimental apparatus.

Resistance and Roughness Coefficients

Leopardi (2004) stated that the tractive force τ produced by the current flow between two sections is proportional to energy gradient, $I = \frac{\Delta H}{L}$ and is expressed as:

$$\tau = \gamma R I \qquad (1)$$

At turbulent flow state, τ depends on density ρ ($\rho = \gamma / g$) mean velocity V, hydraulic radius R and the roughness K, that is:

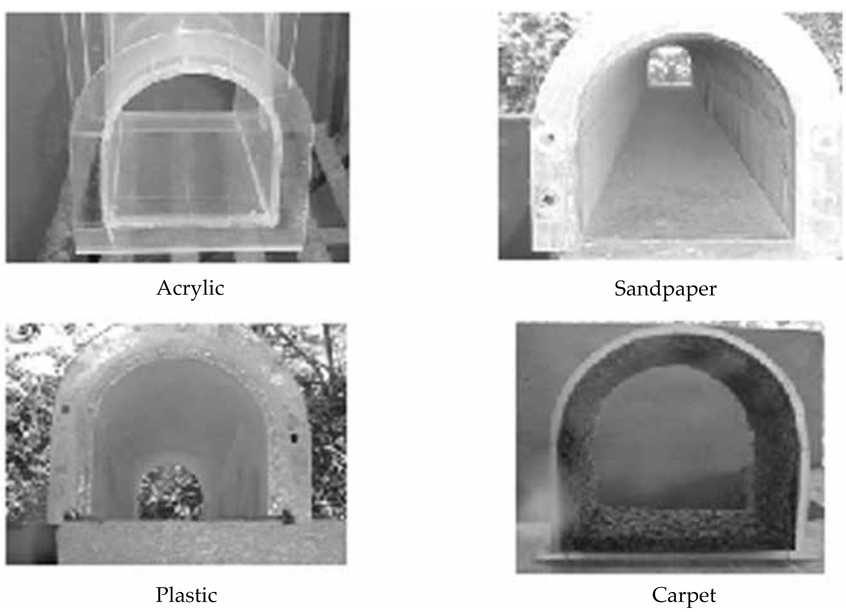

Figure 2. Material used.

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$$\tau = f\left(\rho, \bar{V}, R, K\right) \quad (2)$$

Which, using Buckingham's theorem, can be expresses as:

$$\tau = \frac{\lambda}{8}\rho V^2 \quad (3)$$

Substituting (3) in (1) makes therefore possible to express the energy gradient as:

$$I = \frac{\lambda V^2}{gR} \quad (4)$$

The head loss between two sections can be calculated by solving numerically the momentum equation $(dh/ds) + \Delta = 0$, where H is the trinomial of Bernoulli. Therefore, between two sections the following relation holds:

$$z_1 + h_1 + \alpha_1 \frac{v_1^2}{2g} = z_2 + h_2 + \alpha_2 \frac{v_2^2}{2g} + \Delta \quad (5)$$

where the index 1 and 2 specify the considered section.

In order to determine the hydraulic gradient along the tunnels, piezometers were installed in the sections 6*D* and 28*D* of the models ($L_{1\text{-}2} = 3\ 127$ m). According to the entrance records upstream from section, 6D there are strong effects of contraction and in all the experiments, downstream from section 28*D* there is a clear effect of air entrance; from this section, most of the hypotheses made for the hydraulic functioning as full pipe are no longer valid.

From equation (3):

$$\lambda = \frac{8\tau}{\rho V^2} = \frac{8gRI}{\lambda V^2} \quad (6)$$

Using the above relations makes it actually to calculate I, τ, λ, for different values of discharge in each model.

Also is calculated the absolute roughness with Colebrook-White criteria; for $R_e > 25{,}000$ the Colebrook-White (Yen, 2002) relation in the transition zone is often used,

$$\frac{1}{\sqrt{\lambda}} = -K_1 \log\left(\frac{K_C}{K_2 R} + \frac{K_3}{4R_e\sqrt{\lambda}}\right) \quad (7)$$

For full circular pipe Colebrook (1939), $K_1 = 2.00$, $K_2 = 14.83$, $K_3 = 2.52$, where $R_e = \frac{VR}{v}$ (V = cross sectional average velocity, and R = hydraulic radius). In a vault type section (figure 3).

$$R = \frac{A}{P} = \frac{b^2(4+\pi)}{b(4+\pi)} = b/2;$$

And $4R_e = \frac{4Vb}{2\nu} = \frac{2Vb}{v}$

Then (Eq. (7)) for vault type sections stays:

$$\frac{1}{\sqrt{\lambda}} = -2log\left(\frac{2K_C}{14.83b} + \frac{2.52}{\frac{2Vb}{\nu}\sqrt{\lambda}}\right) \quad (8)$$

Eq. (8) will be used in the analysis.

Hydraulic Models

Models with One Material

Table 1 summarizes the hydraulic parameters calculated for acrylic tunnel model (slopes A1-0.0007, A2-0.001, A3-0.004), for each one, it is reported the slope, the flow rate Q (that was tested in the range $0.01 < Q < 0.025$ m^3/s), the mean velocity $V = Q/A$, the Reynolds number Re in the range $8 \times 10^4 < R_e < 2.4 \times 10^5$ (with kinematic viscosity of water $\upsilon = 0.000001$ m^2/s), then is calculated for each case I, τ, λ, and the absolute roughness K_b for each criteria ($K_{b\,N}$ Nikuardse, K_{bH} Haland, K_{bCH} Churchill and K_{bSw} Swamee, see table A.1), that is used like bottom roughness in the analysis.

Table 2 shows the hydraulic parameters for sand paper (slopes S1 - 0.001, S2 - 0.004, S3 - 0.008), table 3 for plastic (slopes P1 - 0.001, P2 - 0.004, P3 - 0.008) and table 4 for carpet (slopes C1 - 0.001, C2 - 0.004, C3 - 0.008).

For each model, they are reported the same parameters of table 1and the absolute roughness measured with Colebrook-White formula; $K_{mC,i}$ means absolute roughness measured in each material ($K_{mC,S}$ sandpaper, $K_{mC,P}$ plastic, $K_{mC,C}$ carpet).

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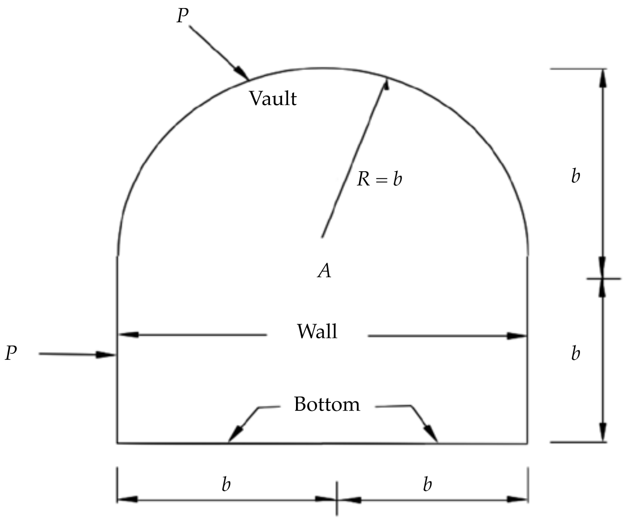

$$A = 2b^2 + \frac{1}{2}\pi b^2 \frac{b^2}{2}(4+\pi) = 0.01579\ \text{m}^2$$

$$P = 4b + \pi b = b\,(4+\pi) = 0.4749\ \text{m}$$

$$P_b = 2b = 0.133\ \text{m}$$

$$P_w = 2b + \pi b = b\,(2+\pi) = 0.34192\ \text{m}$$

$$R = \frac{b^2\,(4+\pi)}{2b\,(4+\pi)} = \frac{b}{2} = 0.3325\ \text{m}$$

Figure 3. Geometry properties of vault type section.

Table 1. Model tests. Acrylic.

S	*Code*	*Q* (m³/s)	*V* (m/s)	*Re*	*I*	τ (kg/m²)	λ	K_{bN} (m)	K_{bH} (m)	K_{bCH} (m)	K_{bSw} (m)
A1 0.0007	A11	0.0192	1.216	162 077	0.0099	0.3278	0.01740	0.000080	0.000027	0.000063	0.000022
	A12	0.0214	1.355	180 648	0.0126	0.4183	0.01787	0.000090	0.000044	0.000075	0.000037
	A13	0.0228	1.444	192 467	0.0143	0.4741	0.01785	0.000089	0.000046	0.000075	0.000040
	A14	0.0245	1.552	206 817	0.0157	0.5208	0.01698	0.000072	0.000030	0.000058	0.000025
	A15	0.0258	1.634	217 791	0.0181	0.6007	0.01766	0.000085	0.000047	0.000072	0.000041
A2 0.001	A21	0.0196	1.241	165 454	0.0104	0.3459	0.01762	0.000084	0.000034	0.000068	0.000028
	A22	0.0214	1.355	180 648	0.0119	0.3972	0.01697	0.000072	0.000024	0.000057	0.000019
	A23	0.0229	1.450	193 311	0.0140	0.4650	0.01735	0.000079	0.000035	0.000065	0.000029
	A24	0.0245	1.552	206 817	0.0153	0.5103	0.01664	0.000065	0.000024	0.000052	0.000019
	A25	0.0257	1.628	216 947	0.0177	0.5887	0.01744	0.000081	0.000042	0.000068	0.000036
A3 0.004	A31	0.0228	1.444	192 467	0.0139	0.4632	0.01744	0.000081	0.000037	0.000066	0.000031
	A32	0.0242	1.533	204 285	0.0160	0.5325	0.01779	0.000088	0.000047	0.000075	0.000041
	A33	0.0261	1.653	220 324	0.0182	0.6064	0.01742	0.000080	0.000042	0.000068	0.000036
	A34	0.0275	1.741	232 142	0.0201	0.6667	0.01725	0.000077	0.000041	0.000065	0.000035
								0.000080	**0.000037**	**0.000066**	**0.000031**

Models with composite roughness

Table 5 shows the hydraulic properties of acrylic-sand paper measured in two roughness model (slopes AS1 - 0.001, AS2 - 0.004, AS - 0.008), table 6, acrylic-plastic (slopes AP1 - 0.001, AP2 - 0.004, AP3 - 0.008), and table 7, acrylic carpet slopes (AC1 - 0.001, AC2 - 0.004, AC3 - 0.008). For each model, it´s calculated like before I, τ, λ, and the measured absolute roughness $K_{mC,i}$ calculated with Colebrook-White criteria ($K_{mC,AS}$ is measured for acrylic-sandpaper, $K_{mC,AP}$ for acrylic plastic and $K_{mC,AC}$ acrylic-carpet). In general, this calculus is taken like the "true value" in the transition zone and it is used for comparison in all over analysis.

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Table 2. Model tests. Sandpaper.

S	*Code*	*Q* (m³/s)	*V* (m/s)	*Re*	*I*	τ (kg/m²)	λ	$K_{mC,S}$ (m)
S1 0,001	S11	0.01700	1.077	143 183	0.0629	2.09213	0.03295	0.00082
	S12	0.01900	1.203	160 028	0.0683	2.27077	0.03204	0.00075
	S13	0.02000	1.267	168 450	0.0743	2.47121	0.03379	0.00090
	S14	0.02100	1.330	176 873	0.0793	2.63773	0.03387	0.00091
	S15	0.02300	1.457	193 718	0.0848	2.82051	0.03377	0.00091
S2 0,004	S21	0.01600	1.013	134 760	0.0579	1.92478	0.03460	0.00096
	S22	0.01800	1.140	151 605	0.0625	2.07912	0.03417	0.00093
	S23	0.01900	1.203	160 028	0.0678	2.25560	0.03467	0.00098
	S24	0.02100	1.330	176 873	0.0734	2.43989	0.03440	0.00096
	S25	0.02200	1.393	185 295	0.0793	2.63684	0.03452	0.00097
	S26	0.02400	1.520	202 140	0.0854	2.84097	0.03428	0.00095
S3 0,008	S31	0.01800	1.140	151 605	0.0622	2.06781	0.03656	0.00116
	S32	0.02000	1.267	168 450	0.0682	2.26757	0.03455	0.00097
	S33	0.02100	1.330	176 873	0.0740	2.46154	0.03622	0.00113
	S34	0.02300	1.457	193 718	0.0794	2.64104	0.03647	0.00116
	S35	0.02400	1.520	202 140	0.0858	2.85435	0.03657	0.00117

Table 3. Model tests. Plastic.

S	*Code*	*Q* (m³/s)	*V* (m/s)	*Re*	*I*	τ (kg/m²)	λ	$K_{mC,P}$ (m)
P1 0.001	P11	0.01500	0.950	126 338	0.01342	0.44636	0.038822	0.00138
	P12	0.01700	1.077	143 183	0.01719	0.57149	0.038698	0.00137
	P13	0.01900	1.203	160 028	0.02136	0.71020	0.038499	0.00136
	P14	0.02000	1.267	168 450	0.02376	0.79011	0.038655	0.00137
	P15	0.02100	1.330	176 873	0.02585	0.85946	0.038139	0.00132
P2 0.004	P21	0.01300	0.823	109 493	0.01026	0.34103	0.039490	0.00145
	P22	0.01500	0.950	126 338	0.01384	0.46014	0.040021	0.00151
	P23	0.01700	1.077	143 183	0.01765	0.58679	0.039734	0.00149
	P24	0.01800	1.140	151 605	0.01946	0.64709	0.039084	0.00142
	P25	0.01900	1.203	160 028	0.02214	0.73605	0.039900	0.00151
	P26	0.02100	1.330	176 873	0.02595	0.86269	0.038282	0.00134
	P27	0.02200	1.393	185 295	0.02885	0.95918	0.038782	0.00139
P3 0.008	P31	0.01800	1.140	151 605	0.01906	0.63388	0.038286	0.00133
	P32	0.01900	1.203	160 028	0.02197	0.73037	0.039592	0.00147
	P33	0.01900	1.203	160 028	0.02174	0.72283	0.039184	0.00143
	P34	0.02000	1.267	168 450	0.02401	0.79821	0.039051	0.00142
	P35	0.02200	1.393	185 295	0.03063	1.01834	0.041174	0.00166

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Table 4. Model tests. Carpet.

S	*Code*	*Q* (m³/s)	*V* (m/s)	*Re*	*I*	τ (kg/m²)	λ	$K_{m,CC}$ (m)
C1 0.001	C11	0.00970	0.614	81 698	0.013243	0.44032	0.091582	0.02191
	C12	0.01080	0.684	90 963	0.017006	0.56546	0.094871	0.02344
	C13	0.01190	0.754	100 228	0.020407	0.67854	0.093769	0.02294
	C14	0.01260	0.798	106 124	0.023445	0.77955	0.096091	0.02402
	C15	0.01350	0.855	113 704	0.026529	0.88208	0.094715	0.02339
	C16	0.01430	0.906	120 442	0.030292	1.00721	0.096389	0.02417
C2 0.004	C21	0.00900	0.570	75 803	0.012071	0.40137	0.096970	0.02440
	C22	0.01000	0.633	84 225	0.015517	0.51595	0.100969	0.02629
	C23	0.01100	0.697	92 648	0.018555	0.61697	0.099783	0.02574
	C24	0.01200	0.760	101 070	0.021956	0.73004	0.099212	0.02548
	C25	0.01300	0.823	109 493	0.025856	0.85970	0.099550	0.02565
	C26	0.01400	0.887	117 915	0.028395	0.94413	0.094266	0.02318
	C27	0.01500	0.950	126 338	0.031977	1.06324	0.092476	0.02236
C3 0,008	C31	0.01000	0.633	84 225	0.015300	0.50874	0.099557	0.02563
	C32	0.01150	0.728	96 859	0.019064	0.63388	0.093797	0.02295
	C33	0.01200	0.760	101 070	0.021785	0.72434	0.098437	0.02512
	C34	0.01350	0.855	113 704	0.025503	0.84797	0.091052	0.02170
	C35	0.01400	0.887	117 915	0.028586	0.95049	0.094901	0.02348
	C36	0.01450	0.918	122 127	0.031624	1.05150	0.097871	0.02486
	C37	0.01500	0.950	126 338	0.035070	1.16609	0.101421	0.02654

Table 5. Model tests. Acrylic-Sandpaper.

S	*Code*	*Q* (m³/s)	*V* (m/s)	*Re*	*I*	τ (kg/m²)	λ	$K_{mC,AS}$ (m)
AS1 0.001	AS11	0.01700	1.077	143 183	0.01206	0.40113	0.02716	0.000404
	AS12	0.01900	1.203	160 028	0.01515	0.50365	0.02730	0.000418
	AS13	0.02000	1.267	168 450	0.01682	0.55943	0.02737	0.000424
	AS14	0.02200	1.393	185 295	0.02023	0.67251	0.02719	0.000417
	AS15	0.02300	1.457	193 718	0.02345	0.77955	0.02884	0.000523
AS2 0.004	AS21	0.01800	1.140	151 605	0.01466	0.48731	0.02943	0.000553
	AS22	0.02000	1.267	168 450	0.01783	0.59284	0.02900	0.000528
	AS23	0.02100	1.330	176 873	0.01937	0.64411	0.02858	0.000502
	AS24	0.02200	1.393	185 295	0.02214	0.73607	0.02976	0.000584
	AS25	0.02400	1.520	202 140	0.02468	0.82050	0.02788	0.000462
AS3 0.008	AS26	0.01800	1.140	151 605	0.01666	0.55397	0.03346	0.000866
	AS31	0.02000	1.267	168 450	0.01888	0.62785	0.03072	0.000650
	AS32	0.02100	1.330	176 873	0.02178	0.72434	0.03214	0.000763
	AS33	0.02300	1.457	193 718	0.02682	0.89169	0.03299	0.000836
	AS34	0.02400	1.520	202 140	0.02863	0.95200	0.03234	0.000784

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Table 6. Model tests. Acrylic-Plastic.

S	*Code*	*Q* (m³/s)	*V* (m/s)	*Re*	*I*	*τ* (kg/m²)	λ	$K_{mC,AP}$ (m)
AP2 0.001	AP11	0.01600	1.013	134 760	0.01299	0.43186	0.033013	0.000822
	AP12	0.01700	1.077	143 183	0.01565	0.52023	0.035227	0.001023
	AP13	0.01900	1.203	160 028	0.01891	0.62879	0.034086	0.000923
	AP14	0.02000	1.267	168 450	0.02177	0.72377	0.035409	0.001047
	AP15	0.02100	1.330	176 873	0.02431	0.80820	0.035864	0.001092
AP3 0.004	AP21	0.01700	1.077	143 183	0.01579	0.52500	0.035550	0.001053
	AP22	0.01800	1.140	151 605	0.01837	0.61094	0.036900	0.001188
	AP23	0.02000	1.267	168 450	0.02109	0.70140	0.034315	0.000946
	AP24	0.02100	1.330	176 873	0.02345	0.77980	0.034604	0.000974
	AP25	0.02300	1.457	193 718	0.03121	1.03761	0.038385	0.001350
AP4 0.008	AP26	0.01800	1.140	151 605	0.01743	0.57964	0.035010	0.001005
	AP31	0.02000	1.267	168 450	0.02111	0.70176	0.034333	0.000948
	AP32	0.02100	1.330	176 873	0.02419	0.80429	0.035690	0.001075
	AP33	0.02200	1.393	185 295	0.02673	0.88872	0.035933	0.001100
	AP34	0.02400	1.520	202 140	0.03174	1.05547	0.035859	0.001096

Table 7. Model tests. Acrylic-Carpet.

S	*Code*	*Q* (m³/s)	*V* (m/s)	*Re*	*I*	τ (kg/m²)	λ	$K_{mC,AC}$ (m)
AC 0.001	AC11	0.0115	0.728	96 859	0.01415	0.47048	0.06962	0.00623
	AC12	0.0124	0.785	104 439	0.01633	0.54285	0.06909	0.00613
	AC13	0.0136	0.861	114 546	0.02018	0.67100	0.07099	0.00651
	AC14	0.0145	0.918	122 127	0.02313	0.76900	0.07158	0.00663
	AC15	0.0154	0.975	129 707	0.02603	0.86549	0.07142	0.00660
	AC21	0.0163	1.032	137 287	0.02893	0.96198	0.07086	0.00649
AC2 0.004	AC22	0.0110	0.697	92 648	0.01293	0.43001	0.06955	0.00621
	AC23	0.0120	0.760	101 070	0.01602	0.53254	0.07237	0.00678
	AC24	0.0130	0.823	109 493	0.01937	0.64411	0.07458	0.00724
	AC25	0.0140	0.887	117 915	0.02146	0.71346	0.07123	0.00656
	AC26	0.0150	0.950	126 338	0.02450	0.81447	0.07084	0.00648
AC3 0.008	AC31	0.0130	0.823	109 493	0.01870	0.62181	0.07200	0.00671
	AC32	0.0140	0.887	117 915	0.02178	0.72434	0.07232	0.00678
	AC33	0.0150	0.950	126 338	0.02519	0.83741	0.07283	0.00689
	AC34	0.0160	1.013	134 760	0.02859	0.95049	0.07266	0.00685
	AC35	0.0170	1.077	143 183	0.03181	1.05754	0.07161	0.00664
	AC36	0.0180	1.140	151 605	0.03471	1.15403	0.06970	0.00627

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Composite Roughness Theoretical Development

In an analysis of Head Loses in Tunnels with two roughness (Elfman, 1993) used Nikuradse Equations; in this article, are used other roughness criteria like Haland, Churchill, and Swamee; also are considered the (16) Eqs. Criteria proposed by Yen (2002) for the analysis of acrylic-plastic and acrylic-carpet (the Reynolds number are very high and turbulence is fully developed).

A tunnel that works with composite roughness is shown in figure 4; has a length L with a cross section that is divided into the corresponding areas A_b and A_w delimited by the perimeters P_b and P_w, has the point of maximum velocity at mid part of tunnel "c" and the contours of equal velocity intersect themselves at right angles.

It is considered that there are two types of roughness K_b (absolute roughness size grain material) at the bottom and K_w in the walls and vault. Total shear force F acting along the tunnel surfaces is equal to the sum of shear force at the bottom ($F_b = LP_b\tau_b$) and shear force at walls and vault ($F_w = LP_w\tau_w$); like $F = F_b + F_w$:

$$\Rightarrow LP\tau = LP_b\tau_b + LP_w\tau_w \quad (9)$$

Considering

$$\tau = \rho V^2\lambda/8;\ \ \tau_b = \rho V_b^2\lambda_b/8;\ \ \tau_w = \rho V_w^2\lambda_w/8 \quad (10)$$

$$PV^2\lambda = P_bV_b^2\lambda_b + P_wV_w^2\lambda_w \quad (11)$$

Maximum velocity V_{max} (point "c", figure4) is equal to maximum velocity V_{maxb} of the tunnel bottom and the maximum velocity at walls and vault $V_{max\,w}$:

$$V_{max} = V_{maxb} = V_{max\,w} \quad (12)$$

From Nikuradse (1933) analysis:

$$V_{max} = V + 3{,}75\left(\frac{\tau}{\rho}\right)^{1/2} \quad (13)$$

For the experimental analysis, form Eq. (12) and (13) in Eq. (11):

$$\frac{P}{\left(\lambda^{-1/2}+1{,}326\right)^2} = \frac{P_b}{\left(\lambda_b^{-1/2}+1{,}326\right)^2} + \frac{P_w}{\left(\lambda_w^{-1/2}+1{,}326\right)^2} \quad (14)$$

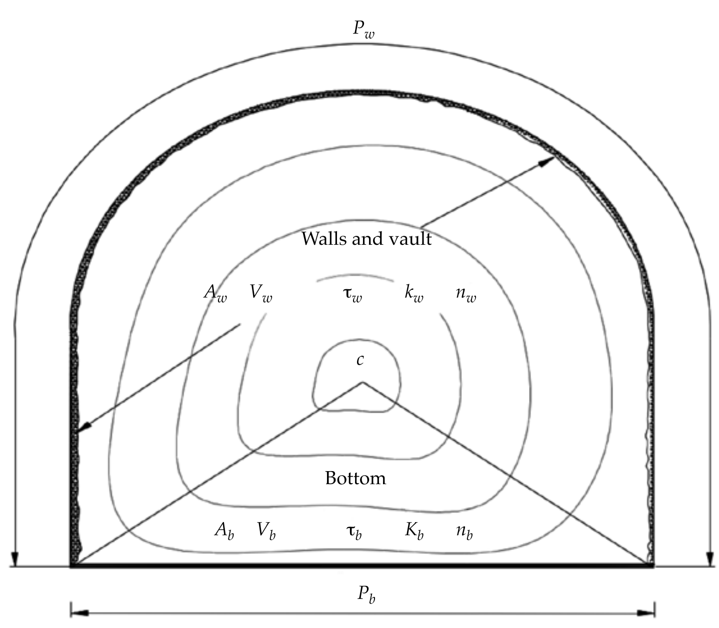

Figure 4. Cross section of the tunnel and contours of equal velocity.

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Head losses at the bottom are equal to those at the walls and vault:

$$\Delta h_b = \Delta h_w \Rightarrow \quad \frac{L_b V_b^2 \lambda_b}{2gR_b} = \frac{L_w V_w^2 \lambda_w}{2gR_w} \tag{15}$$

$$\therefore \frac{V_b^2 \lambda_b}{\frac{A_b}{P_b}} = \frac{V_w^2 \lambda_w}{\frac{A_w}{P_w}}; \frac{P_b V_b^2 \lambda_b}{A_b} = \frac{P_w V_w^2 \lambda_w}{A_w} \tag{16}$$

Like: $V_b = \frac{V_{\max b}}{\left(1+1.326\lambda_b^{1/2}\right)}$ and $V_w = \frac{V_{\max w}}{\left(1+1.326\lambda_w^{1/2}\right)}$

$$\Rightarrow \frac{P_b}{A_b\left(\lambda_b^{-1/2}+1,326\right)^2} = \frac{P_w}{A_w\left(\lambda_w^{-1/2}+1,326\right)^2} \tag{17}$$

There are two scenarios of analysis:

Scenario I. If the total roughness factor λ is experimentally known, and also the material of the tunnel bottom K_b, form Eq. (17):

$$\frac{A_w P_b}{A_b\left(\lambda_b^{-1/2}+1,326\right)^2} = \frac{P_w}{\left(\lambda_w^{-1/2}+1,326\right)^2} \tag{18}$$

In Eq. (11):

$$\frac{P}{\left(\lambda^{-1/2}+1,326\right)^2} = \frac{P_b}{\left(\lambda_b^{-1/2}+1,326\right)^2}\left(1+\frac{A_w}{A_b}\right) \tag{19}$$

Since $A = A_b + A_w$:

$$\frac{P}{A\left(\lambda^{-1/2}+1,326\right)^2} = \frac{P_b}{A_b\left(\lambda_b^{-1/2}+1,326\right)^2} \tag{20}$$

Tables 8, 9, 10 and 11 summarizes the calculus of the variables showed in this first scenario A_b, A_w, λ_b, λ_w, $K_{w,i}$; $K_{w,i}$ means the absolute roughness in walls and vault (i, is N for Nikuradse, H for Haland, Ch for Churchill and S for Swamee).

Like first comparison, $K_{w,i}$ calculated with Nikuradse, Haland, Churchill and Swamee criteria is compared with measured sandpaper model (figure 5). In general the data fits very well the behavior of measured values (table 3), and calculated values (tables 8, 9, 10, and 11).

Table 8. Calculus of Acrylic-sandpaper (Nikuradse criteria), Scenario I.

S	Code	Q (m³/s)	λ	V (m/s)	Re	A_b (m²)	A_w (m²)	λb	λw	$K_{w,N}$ (m)
0.001	AS11	0.017	0.027162	1.077	143183	0.00302	0.0128	0.0190	0.03052	0.000762
	AS12	0.019	0.027302	1.203	160028	0.00301	0.0128	0.0191	0.03071	0.000778
	AS13	0.020	0.027369	1.267	168450	0.00300	0.0128	0.0191	0.03081	0.000787
	AS14	0.022	0.027191	1.393	185295	0.00302	0.0128	0.0190	0.03056	0.000765
	AS15	0.023	0.028838	1.457	193718	0.00288	0.0129	0.0193	0.03283	0.000976
0.004	AS21	0.018	0.0294330	1.140	151605	0.00283	0.0130	0.0193	0.03366	0.001059
	AS22	0.020	0.0290041	1.267	168450	0.00286	0.0129	0.0193	0.03306	0.000998
	AS23	0.021	0.028582	1.330	176873	0.00290	0.0129	0.0192	0.03248	0.000941
	AS24	0.022	0.029761	1.393	185295	0.00280	0.0130	0.0194	0.03411	0.001106
	AS25	0.024	0.027876	1.520	202140	0.00296	0.0128	0.0191	0.03151	0.000849
0.008	AS26	0.018	0.032002	1.140	151605	0.00264	0.0131	0.0197	0.03723	0.001463
	AS31	0.020	0.03072	1.267	168450	0.00273	0.0131	0.0195	0.03544	0.001252
	AS32	0.021	0.032143	1.330	176873	0.00263	0.0132	0.0197	0.03743	0.001487
	AS33	0.023	0.032987	1.457	193718	0.00258	0.0132	0.0198	0.03861	0.001637
	AS34	0.024	0.032344	1.520	202140	0.00262	0.0132	0.0197	0.03771	0.001522

Nikuradse *Kb* = 0.000080.

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Table 9. Calculus of acrylic-sandpaper (Haland criteria), Scenario I.

S	Code	Q (m³/s)	λ	V (m/s)	Re	A_b (m²)	A_w (m²)	λb	λw	$K_{w,H}$ (m)
0.001	AS11	0.017	0.027162	1.077	143 183	0.00315	0.0126	0.01812	0.03093	0.000621
	AS12	0.019	0.027302	1.203	160 028	0.00310	0.0127	0.01785	0.03126	0.000649
	AS13	0.020	0.027369	1.267	168 450	0.00307	0.0127	0.01774	0.03141	0.000662
	AS14	0.022	0.027191	1.393	185 295	0.00306	0.0127	0.01753	0.03124	0.000653
	AS15	0.023	0.028838	1.457	193 718	0.00290	0.0129	0.01743	0.03367	0.000843
0.004	AS21	0.018	0.0294330	1.140	151 605	0.00293	0.0129	0.01798	0.03428	0.000886
	AS22	0.020	0.0290041	1.267	168 450	0.00293	0.0129	0.01774	0.03377	0.000847
	AS23	0.021	0.028582	1.330	176 873	0.00295	0.0128	0.01763	0.03321	0.000802
	AS24	0.022	0.029761	1.393	185 295	0.00284	0.0130	0.01753	0.03497	0.000952
	AS25	0.024	0.027876	1.520	202 140	0.00297	0.0128	0.01735	0.03232	0.000736
0.008	AS26	0.018	0.032002	1.140	151 605	0.00274	0.0131	0.01798	0.03803	0.001232
	AS31	0.020	0.03072	1.267	168 450	0.00280	0.0130	0.01774	0.03627	0.001066
	AS32	0.021	0.032143	1.330	176 873	0.00268	0.0131	0.01763	0.03841	0.001275
	AS33	0.023	0.032987	1.457	193 718	0.00260	0.0132	0.01743	0.03974	0.001416
	AS34	0.024	0.032344	1.520	202 140	0.00263	0.0132	0.01735	0.03884	0.001323

Haland *Kb* = 0.000037.

Table 10. Calculus of acrylic-sandpaper (Churchill criteria), Scenario I.

S	Code	Q (m³/s)	λ	V (m/s)	Re	A_b (m²)	A_w (m²)	λb	λw	$K_{w,CH}$ (m)
0.001	AS11	0.017	0.027162	1.077	143 183	0.00291	0.0129	0.0167	0.03160	0.000740
	AS12	0.019	0.027302	1.203	160 028	0.00290	0.0129	0.0167	0.03180	0.000758
	AS13	0.020	0.027369	1.267	168 450	0.00289	0.0129	0.0167	0.03190	0.000766
	AS14	0.022	0.027191	1.393	185 295	0.00291	0.0129	0.0167	0.03164	0.000747
	AS15	0.023	0.028838	1.457	193 718	0.00277	0.0130	0.0167	0.03403	0.000947
0.004	AS21	0.018	0.0294330	1.140	151 605	0.00273	0.0131	0.0167	0.03490	0.001021
	AS22	0.020	0.0290041	1.267	168 450	0.00276	0.0130	0.0167	0.03428	0.000966
	AS23	0.021	0.028582	1.330	176 873	0.00279	0.0130	0.0167	0.03366	0.000913
	AS24	0.022	0.029761	1.393	185 295	0.00270	0.0131	0.0167	0.03538	0.001069
	AS25	0.024	0.027876	1.520	202 140	0.00285	0.0129	0.0167	0.03263	0.000828
0.008	AS26	0.018	0.032002	1.140	151 605	0.00255	0.0132	0.0167	0.03867	0.001396
	AS31	0.020	0.03072	1.267	168 450	0.00263	0.0132	0.0167	0.03678	0.001202
	AS32	0.021	0.032143	1.330	176 873	0.00254	0.0133	0.0167	0.03888	0.001420
	AS33	0.023	0.032987	1.457	193 718	0.00249	0.0133	0.0167	0.04012	0.001559
	AS34	0.024	0.032344	1.520	202 140	0.00253	0.0133	0.0167	0.03917	0.001454

Churchill *Kb* = 0.000066.

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Table 11. Calculus of acrylic-sandpaper (Swamee criteria), Scenario I.

S	*Code*	*Q* (m³/s)	λ	*V* (m/s)	*Re*	A_b (m²)	A_w (m²)	λb	λw	$K_{w,S}$ (m)
0.001	AS11	0.017	0.027162	1.077	143 183	0.00253	0.0133	0.01416	0.03277	0.000788
	AS12	0.019	0.027302	1.203	160 028	0.00252	0.0133	0.01416	0.03298	0.000812
	AS13	0.020	0.027369	1.267	168 450	0.00252	0.0133	0.01416	0.03307	0.000822
	AS14	0.022	0.027191	1.393	185 295	0.00253	0.0133	0.01416	0.03281	0.000805
	AS15	0.023	0.028838	1.457	193 718	0.00241	0.0134	0.01416	0.03523	0.001020
0.004	AS21	0.018	0.0294330	1.140	151 605	0.00237	0.0134	0.01416	0.03611	0.001091
	AS22	0.020	0.0290041	1.267	168 450	0.00240	0.0134	0.01416	0.03548	0.001037
	AS23	0.021	0.028582	1.330	176 873	0.00243	0.0134	0.01416	0.03486	0.000981
	AS24	0.022	0.029761	1.393	185 295	0.00235	0.0134	0.01416	0.03660	0.001149
	AS25	0.024	0.027876	1.520	202 140	0.00248	0.0133	0.01416	0.03382	0.000895
0.008	AS26	0.018	0.032002	1.140	151 605	0.00222	0.0136	0.01416	0.03992	0.001488
	AS31	0.0200	0.03072	1.267	168 450	0.00229	0.0135	0.01416	0.03801	0.001288
	AS32	0.0210	0.032143	1.330	176 873	0.00221	0.0136	0.01416	0.04012	0.001520
	AS33	0.0230	0.032987	1.457	193 718	0.00216	0.0136	0.01416	0.04138	0.001668
	AS34	0.0240	0.032344	1.520	202 140	0.00220	0.0136	0.01416	0.04042	0.001560

Swamee Kb = 0.000031.

Scenario II. If geometry, absolute roughness materials (K_b, K_w) and influence perimeters (P_b, P_w) are known, it is desired to calculate total roughness factor λ of the model tunnel.

The absolute roughness values $K_{w,i}$ calculated in Scenario I (tables 8, 9, 10 and 11) are used in order to calculate λ and the value of $K_{predC,i}$ with Eq. (17) (and Eq. 14). This predicted values of absolute roughness calculated with Colebrook's formula, let us to compare the measured values (tables 5, 6, 7 and the predicted values with the theoretical formulation).

Table 12 show the results for Nikuradse, table 13 for Haland, table 14 for Churchill and table 15 for Swamee criteria, using the results obtained from scenario I.

Statistical Comparison

The statistical comparison of any friction factor equation with the Colebrook's equation can be done with the following procedure.

- Calculate the friction factor $K_{pred,i}$ by the criteria selected (Nikradse, Haland, Churchill, Swamee).
- Calculate the friction factor value $K_{mC,i}$ measured with the Colebrook's equation.
- Calculate the following parameters:

Mean relative error

$$meanREK = \frac{1}{n}\sum_{i=1}^{n}\frac{\left|K_{mC,i} - K_{pred,i}\right|}{K_{mC,i}} \quad (21)$$

Maximal positive error

$$\max RE^{+} = \max\left(\frac{K_{mC,i} - K_{pred,i}}{K_{mC,i}}\right) \quad (22)$$

Maximal negative error

$$\max RE^{-} = \max\left(\frac{K_{pred,i} - K_{mC,i}}{K_{mC,i}}\right) \quad (23)$$

Correlation ratio

$$\Theta = \sqrt{1 - \frac{\sum_{i=1}^{n}\left(K_{mC,i} - K_{pred,i}\right)^2}{\sum_{i=1}^{n}\left(K_{mC,i} - \bar{K}_{m}\right)^2}} \quad (24)$$

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Table 12. Calculus of acrylic-sandpaper (Nikuradse criteria), scenario II.

S	*Code*	*Q* (m^3/s)	$K_{w,N}$ (m)	*V* (m/s)	*Re*	A_b (m^2)	A_w (m^2)	λb	λw	λ	$K_{pred,N}$ (m)
	AS11	0.017	0.000762	1.077	143 183	0.003248	0.0125	0.0187	0.0307	0.02726	0.0004616
	AS12	0.019	0.000778	1.203	160 028	0.003237	0.0126	0.0187	0.0309	0.02740	0.0004700
0.001	AS13	0.020	0.000787	1.267	168 450	0.003232	0.0126	0.0187	0.0310	0.02747	0.0004741
	AS14	0.022	0.000765	1.393	185 295	0.003246	0.0125	0.0187	0.0307	0.02729	0.0004634
	AS15	0.023	0.000976	1.457	193 718	0.003118	0.0127	0.0189	0.0330	0.02894	0.0005674
	AS21	0.018	0.001059	1.140	151 605	0.003075	0.0127	0.0190	0.0339	0.02954	0.0006078
	AS22	0.020	0.000998	1.267	168 450	0.003106	0.0127	0.0189	0.0333	0.029111	0.0005785
0.004	AS23	0.021	0.000941	1.330	176 873	0.003137	0.0127	0.0189	0.0327	0.02869	0.0005505
	AS24	0.022	0.001106	1.393	185 295	0.003052	0.0127	0.0190	0.0343	0.02987	0.0006309
	AS25	0.024	0.000849	1.520	202 140	0.003191	0.0126	0.0188	0.0317	0.027977	0.0005053
	AS26	0.018	0.001463	1.140	151 605	0.002906	0.0129	0.0192	0.0375	0.03213	0.0008005
	AS31	0.0200	0.001252	1.267	168 450	0.002988	0.0128	0.0191	0.0357	0.030835	0.0007005
0.008	AS32	0.0210	0.001487	1.330	176 873	0.002898	0.0129	0.0192	0.0377	0.03227	0.0008119
	AS33	0.0230	0.001637	1.457	193 718	0.002848	0.0129	0.0193	0.0389	0.03312	0.0008820
	AS34	0.0240	0.001522	1.520	202 140	0.002886	0.0129	0.0193	0.0380	0.032475	0.0008284

Nikuradse *Kb* = 0.000080.

Table 13. Calculus of acrylic-sandpaper (Haland criteria), scenario II.

S	*Code*	*Q* (m^3/s)	$K_{w,H}$ (m)	*V* (m/s)	*Re*	A_b (m^2)	A_w (m^2)	λb	λw	λ	$K_{pred,H}$ (m)
	AS11	0.017	0.000621	1.077	143 183	0.003222	0.0126	0.01812	0.03042	0.02688	0.000391
	AS12	0.019	0.000649	1.203	160 028	0.003169	0.0126	0.01785	0.03073	0.02702	0.000403
0.001	AS13	0.020	0.000662	1.267	168 450	0.003145	0.0126	0.01774	0.03088	0.02708	0.000409
	AS14	0.022	0.000653	1.393	185 295	0.003130	0.0127	0.01753	0.03072	0.02691	0.000403
	AS15	0.023	0.000843	1.457	193 718	0.002972	0.0128	0.01743	0.03310	0.02853	0.000504
	AS21	0.018	0.000886	1.140	151 605	0.003002	0.0128	0.01798	0.03370	0.02912	0.000533
	AS22	0.020	0.000847	1.267	168 450	0.003002	0.0128	0.01774	0.03320	0.028696	0.000509
0.004	AS23	0.021	0.000802	1.330	176 873	0.003022	0.0128	0.01763	0.03265	0.02828	0.000484
	AS24	0.022	0.000952	1.393	185 295	0.002911	0.0129	0.01753	0.03437	0.02944	0.000563
	AS25	0.024	0.000736	1.520	202 140	0.003041	0.0127	0.01735	0.03177	0.027581	0.000445
	AS26	0.018	0.001232	1.140	151 605	0.002808	0.0130	0.01798	0.03738	0.03165	0.000717
	AS31	0.0200	0.001066	1.267	168 450	0.002867	0.0129	0.01774	0.03564	0.030385	0.000626
0.008	AS32	0.0210	0.001275	1.330	176 873	0.002751	0.0130	0.01763	0.03774	0.03179	0.000734
	AS33	0.0230	0.001416	1.457	193 718	0.002669	0.0131	0.01743	0.03905	0.03262	0.000804
	AS34	0.0240	0.001323	1.520	202 140	0.002700	0.0131	0.01735	0.03816	0.031988	0.000754

Haland *Kb* = 0.000037.

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Table 14. Calculus of acrylic-sandpaper (Churchill criteria), scenario II.

S	Code	Q (m³/s)	$K_{w,Ch}$ (m)	V (m/s)	Re	A_b (m²)	A_w (m²)	λb	λw	λ	$K_{pred,Ch}$ (m)
0.001	AS11	0.017	0.000740	1.077	143 183	0.002968	0.0128	0.01666	0.031355	0.02666	0.000408
	AS12	0.019	0.000758	1.203	160 028	0.002954	0.0128	0.01666	0.031584	0.02686	0.000422
	AS13	0.020	0.000766	1.267	168 450	0.002947	0.0128	0.01666	0.031693	0.02696	0.000428
	AS14	0.022	0.000747	1.393	185 295	0.002962	0.0128	0.01666	0.031449	0.02681	0.000421
	AS15	0.023	0.000947	1.457	193 718	0.002822	0.0130	0.01666	0.033872	0.02852	0.000526
0.004	AS21	0.018	0.001021	1.140	151 605	0.002777	0.0130	0.01666	0.034708	0.02909	0.000559
	AS22	0.020	0.000966	1.267	168 450	0.002810	0.0130	0.01666	0.034094	0.028670	0.000533
	AS23	0.021	0.000913	1.330	176 873	0.002843	0.0129	0.01666	0.033482	0.02825	0.000507
	AS24	0.022	0.001069	1.393	185 295	0.002751	0.0130	0.01666	0.035224	0.02948	0.000589
	AS25	0.024	0.000828	1.520	202 140	0.002901	0.0129	0.01666	0.032467	0.027563	0.000466
0.008	AS26	0.018	0.001396	1.140	151 605	0.002594	0.0132	0.01666	0.038502	0.03171	0.000750
	AS31	0.0200	0.001202	1.267	168 450	0.002681	0.0131	0.01666	0.036617	0.030426	0.000655
	AS32	0.0210	0.001420	1.330	176 873	0.002584	0.0132	0.01666	0.038731	0.03191	0.000768
	AS33	0.0230	0.001559	1.457	193 718	0.002531	0.0133	0.01666	0.039992	0.03280	0.000841
	AS34	0.0240	0.001454	1.520	202 140	0.002571	0.0134	0.01666	0.039045	0.032152	0.000789

Churchill *Kb* = 0.000066.

Table 15. Calculus of acrylic-sandpaper (Swamee criteria), scenario II.

S	Code	Q (m³/s)	$K_{w,S}$ (m)	V (m/s)	Re	A_b (m²)	A_w (m²)	λb	λw	λ	$K_{pred,S}$ (m)
0.001	AS11	0.017	0.000788	1.077	143 183	0.002607	0.0132	0.01416	0.03198	0.02671	0.000366
	AS12	0.019	0.000812	1.203	160 028	0.002591	0.0132	0.01416	0.03227	0.02691	0.000383
	AS13	0.020	0.000822	1.267	168 450	0.002584	0.0132	0.01416	0.03240	0.02700	0.000391
	AS14	0.022	0.000805	1.393	185 295	0.002595	0.0132	0.01416	0.03219	0.02689	0.000389
	AS15	0.023	0.001020	1.457	193 718	0.002468	0.0133	0.01416	0.03470	0.02854	0.000492
0.004	AS21	0.018	0.001091	1.140	151 605	0.002432	0.0134	0.01416	0.03547	0.02911	0.000518
	AS22	0.020	0.001037	1.267	168 450	0.002459	0.0133	0.01416	0.03488	0.028695	0.000496
	AS23	0.021	0.000981	1.330	176 873	0.002489	0.0133	0.01416	0.03427	0.02828	0.000471
	AS24	0.022	0.001149	1.393	185 295	0.002404	0.0134	0.01416	0.03607	0.02951	0.000555
	AS25	0.024	0.000895	1.520	202 140	0.002538	0.0133	0.01416	0.03327	0.027583	0.000434
0.008	AS26	0.018	0.001488	1.140	151 605	0.002267	0.0135	0.01416	0.03936	0.03174	0.000709
	AS31	0.0200	0.001288	1.267	168 450	0.002344	0.0134	0.01416	0.03747	0.030458	0.000618
	AS32	0.0210	0.001520	1.330	176 873	0.002257	0.0135	0.01416	0.03965	0.03193	0.000732
	AS33	0.0230	0.001668	1.457	193 718	0.002208	0.0136	0.01416	0.04096	0.03282	0.000808
	AS34	0.0240	0.001560	1.520	202 140	0.002243	0.0135	0.01416	0.04001	0.032176	0.000757

Swamee *Kb* = 0.000031.

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Standard deviation

$$\Delta_{SD}=\sqrt{\frac{\sum_{i=1}^{n}\left(\frac{K_{mC,i}-K_{pred,i}}{K_{mC,i}}\right)^2}{n}} \quad (25)$$

Mean variation

$$\Delta_{m,i}=\frac{\bar{K}_m-\bar{K}_{predC,i}}{\bar{K}_m} \quad (26)$$

Where $\bar{K}_m$ is the average value of $K_{mC,i}$ for the complete set of values:

$$\bar{K}_m=\sum_{i=i}^{n}\left(\frac{K_{mC,i}}{n}\right) \quad (27)$$

In the table 16 is made a comparison with the mean relative, maximal positive, maximal negative and mean variation error for each criteria (Nikuradse, Haland, Churchill, and Swamee).

The maximal positive relative error maxRE^+ = 0.0936 is for Swamee criteria; the maximum negative error belongs to the Nikuradse criteria maxRE^- = 0.144.

In table 20 is shown a summary of this calculus; in which are shown the correlation ratio, standard deviation and mean variation of the error.

The statistical analysis, show that the best correlation ratio and the minimum standard deviation is for Churchill criteria.

The minimum difference in the in the mean value $\Delta_m=\frac{K_{mC,i}-K_{pred,i}}{K_{meas}}$ is for Haland criteria (-0.03665) although the obtained for Churchill (0.05517) and Swamee (0.05515) are very similar.

Conclusions

It was proposed the Eq. (8) that is a Colebrook-White form, and let to determine the absolute roughness in this kind of tunnels geometries; even when it is applied to the composite roughness. In addition, it was reviewed other

Table 16. Calculus of acrylic-sandpaper (Nikuradse criteria).

	Q **(m³/s)**	***V*** **(m/s)**	***K*meas**	***K*nik**	$\frac{\lvert K_{mC,i}-K_{pred,i}\rvert}{K_{mC,i}}$	maxRE^+	maxRE^-	$(K_m-K_c)^2$	$(K_m-\bar{K}_m)^2$	$\left(\frac{K_m-K_c}{K_m}\right)^2$
0.0010	0.0170	1.077	0.000404	0.000462	0.144	-0.144	0.144	3.363E-09	1.629E-07	2.064E-02
	0.0190	1.203	0.000418	0.000470	0.126	-0.126	0.126	2.754E-09	1.743E-07	1.580E-02
	0.0200	1.267	0.000424	0.000474	0.118	-0.118	0.118	2.515E-09	1.797E-07	1.399E-02
	0.0220	1.393	0.000417	0.000463	0.111	-0.111	0.111	2.127E-09	1.741E-07	1.222E-02
	0.0230	1.457	0.000523	0.000567	0.085	-0.085	0.085	1.988E-09	2.734E-07	7.273E-03
0.0040	0.0180	1.140	0.000553	0.000608	0.100	-0.100	0.100	3.027E-09	3.056E-07	9.904E-03
	0.0200	1.267	0.000528	0.000579	0.095	-0.095	0.095	2.522E-09	2.791E-07	9.036E-03
	0.0210	1.330	0.000502	0.000550	0.096	-0.096	0.096	2.315E-09	2.524E-07	9.174E-03
	0.0220	1.393	0.000584	0.000631	0.080	-0.080	0.080	2.163E-09	3.416E-07	6.333E-03
	0.0240	1.520	0.000462	0.000505	0.093	-0.093	0.093	1.835E-09	2.138E-07	8.583E-03
0.0080	0.0180	1.140	0.000745	0.000800	0.075	-0.075	0.075	3.094E-09	5.548E-07	5.576E-03
	0.0200	1.267	0.000650	0.000701	0.078	-0.078	0.078	2.561E-09	4.224E-07	6.064E-03
	0.0210	1.330	0.000763	0.000812	0.065	-0.065	0.065	2.424E-09	5.817E-07	4.167E-03
	0.0230	1.457	0.000836	0.000882	0.055	-0.055	0.055	2.150E-09	6.983E-07	3.078E-03
	0.0240	1.520	0.000784	0.000828	0.057	-0.057	0.057	1.981E-09	6.145E-07	3.225E-03
Mean			**0.000573**	**0.000622**	**0.092**	**-0.055**	**0.144**			
Θ									**0.99647**	
Δ_{SD}										**0.009004**

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Table 17. Calculus of acrylic-sandpaper (Haland criteria).

S	*Q* (m³/s)	*V* (m/s)	*K*meas	*K*hal	$\frac{\lvert K_{mC,i}-K_{pred,i}\rvert}{K_{mC,i}}$	max*RE*$^+$	max*RE*$^-$	$(K_m-K_c)^2$	$(K_m-\overline{K}_m)^2$	$\left(\frac{K_m-K_c}{K_m}\right)^2$
0.0010	0.0170	1.077	0.000404	0.000391	0.03230	0.03230	-0.03230	1.700E-10	1.629E-07	2.074E-02
	0.0190	1.203	0.000418	0.000403	0.03388	0.03388	-0.03388	2.001E-10	1.743E-07	1.148E-03
	0.0200	1.267	0.000424	0.000409	0.03452	0.03452	-0.03452	2.142E-10	1.797E-07	1.192E-03
	0.0220	1.393	0.000417	0.000403	0.03527	0.03527	-0.03527	2.167E-10	1.741E-07	1.244E-03
	0.0230	1.457	0.000523	0.000504	0.03700	0.03700	-0.03700	3.743E-10	2.734E-07	1.369E-03
0.0040	0.0180	1.140	0.000553	0.000533	0.03561	0.03561	-0.03561	3.876E-10	3.056E-07	1.268E-03
	0.0200	1.267	0.000528	0.000509	0.03611	0.03611	-0.03611	3.639E-10	2.791E-07	1.304E-03
	0.0210	1.330	0.000502	0.000484	0.03617	0.03617	-0.03617	3.302E-10	2.524E-07	1.308E-03
	0.0220	1.393	0.000584	0.000563	0.03717	0.03717	-0.03717	4.719E-10	3.416E-07	1.382E-03
	0.0240	1.520	0.000462	0.000445	0.03668	0.03668	-0.03668	2.877E-10	2.138E-07	1.345E-03
0.0080	0.0180	1.140	0.000745	0.000717	0.03684	0.03684	-0.03684	7.529E-10	5.548E-07	1.357E-03
	0.0200	1.267	0.000650	0.000626	0.03699	0.03699	-0.03699	5.780E-10	4.224E-07	1.369E-03
	0.0210	1.330	0.000763	0.000734	0.03755	0.03755	-0.03755	8.203E-10	5.817E-07	1.410E-03
	0.0230	1.457	0.000836	0.000804	0.03793	0.03793	-0.03793	1.005E-09	6.983E-07	1.439E-03
	0.0240	1.520	0.000784	0.000754	0.03807	0.03807	-0.03807	8.906E-10	6.145E-07	1.449E-03
Mean			**0.000573**	**0.000552**	**0.03614**	**0.03807**	**-0.03230**			
Θ									**0.999324**	
Δ_{SD}										**0.00262**

Table 18. Calculus of acrylic-sandpaper (Churchill criteria).

S	*Q* (m³/s)	*V* (m/s)	*K*meas	*K*chur	$\frac{\lvert K_{mC,i}-K_{pred,i}\rvert}{K_{mC,i}}$	max*RE*$^+$	max*RE*$^-$	$(K_m-K_c)^2$	$(K_m-\overline{K}_m)^2$	$\left(\frac{K_m-K_c}{K_m}\right)^2$
0.0010	0.0170	1.077	0.000404	0.000408	0.01181	-0.01181	0.01181	2.272E-11	1.629E-07	1.395E-04
	0.0190	1.203	0.000418	0.000422	0.01032	-0.01032	0.01032	1.857E-11	1.743E-07	1.065E-04
	0.0200	1.267	0.000424	0.000428	0.00976	-0.00976	0.00976	1.712E-11	1.797E-07	9.526E-05
	0.0220	1.393	0.000417	0.000421	0.00918	-0.00918	0.00918	1.468E-11	1.741E-07	8.430E-05
	0.0230	1.457	0.000523	0.000526	0.00526	-0.00526	0.00526	7.565E-12	2.734E-07	2.767E-05
0.0040	0.0180	1.140	0.000553	0.000559	0.01185	-0.01185	0.01185	4.289E-11	3.056E-07	1.403E-04
	0.0200	1.267	0.000528	0.000533	0.00978	-0.00978	0.00978	2.667E-11	2.791E-07	9.556E-05
	0.0210	1.330	0.000502	0.000507	0.01009	-0.01009	0.01009	2.570E-11	2.524E-07	1.019E-04
	0.0220	1.393	0.000584	0.000589	0.00735	-0.00735	0.00735	1.845E-11	3.416E-07	5.402E-05
	0.0240	1.520	0.000462	0.000466	0.00857	-0.00857	0.00857	1.569E-11	2.138E-07	7.339E-05
0.0080	0.0180	1.140	0.000745	0.000750	0.00674	-0.00674	0.00674	2.518E-11	5.548E-07	4.538E-05
	0.0200	1.267	0.000650	0.000655	0.00712	-0.00712	0.00712	2.144E-11	4.224E-07	5.076E-05
	0.0210	1.330	0.000763	0.000768	0.00646	-0.00646	0.00646	2.428E-11	5.817E-07	4.175E-05
	0.0230	1.457	0.000836	0.000841	0.00622	-0.00622	0.00622	2.699E-11	6.983E-07	3.864E-05
	0.0240	1.520	0.000784	0.000789	0.00635	-0.00635	0.00635	2.479E-11	6.145E-07	4.034E-05
Mean			**0.000573**	**0.00054139**	**0.00846**	**-0.00526**	**0.01185**			
Θ									**0.9999682**	
Δ_{SD}										**7.5682E-05**

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Table 19. Calculus of acrylic-sandpaper (Swamee criteria).

S	Q (m³/s)	V (m/s)	*K*meas	*K*swam	$\frac{\lvert K_{mC,i}-K_{pred,i}\rvert}{K_{mC,i}}$	max*RE*⁺	max*RE*⁻	$\left(K_m-K_c\right)^2$	$\left(K_m-\overline{K}_m\right)^2$	$\left(\frac{K_m-K_c}{K_m}\right)^2$
0.0010	0.0170	1.077	0.000404	0.000366	0.0936	0.0936	-0.0936	1.426E-09	2.864E-08	8.75E-03
	0.0190	1.203	0.000418	0.000383	0.0818	0.0818	-0.0818	1.168E-09	2.412E-08	6.70E-03
	0.0200	1.267	0.000424	0.000391	0.0769	0.0769	-0.0769	1.063E-09	2.217E-08	5.92E-03
	0.0220	1.393	0.000417	0.000389	0.0669	0.0669	-0.0669	7.796E-10	2.419E-08	4.48E-03
	0.0230	1.457	0.000523	0.000492	0.0582	0.0582	-0.0582	9.255E-10	2.498E-09	3.39E-03
0.0040	0.0180	1.140	0.000553	0.000518	0.0631	0.0631	-0.0631	1.217E-09	4.004E-10	3.98E-03
	0.0200	1.267	0.000528	0.000496	0.0616	0.0616	-0.0616	1.059E-09	1.983E-09	3.80E-03
	0.0210	1.330	0.000502	0.000471	0.0619	0.0619	-0.0619	9.667E-10	4.968E-09	3.83E-03
	0.0220	1.393	0.000584	0.000555	0.0506	0.0506	-0.0506	8.759E-10	1.343E-10	2.56E-03
	0.0240	1.520	0.000462	0.000434	0.0609	0.0609	-0.0609	7.932E-10	1.219E-08	3.71E-03
0.0080	0.0180	1.140	0.000745	0.000709	0.0480	0.0480	-0.0480	1.277E-09	2.960E-08	2.30E-03
	0.0200	1.267	0.000650	0.000618	0.0498	0.0498	-0.0498	1.048E-09	5.940E-09	2.48E-03
	0.0210	1.330	0.000763	0.000732	0.0400	0.0400	-0.0400	9.314E-10	3.604E-08	1.60E-03
	0.0230	1.457	0.000836	0.000808	0.0329	0.0329	-0.0329	7.540E-10	6.907E-08	1.08E-03
	0.0240	1.520	0.000784	0.000757	0.0337	0.0337	-0.0337	6.984E-10	4.454E-08	1.14E-03
Mean			**0.000573**	**0.0005414**	**0.0587**	**0.0936**	**-0.0329**			
Θ									**0.975248717**	
Δ_{SD}										**0.003714213**

Table 20. Summary of statistical analysis.

Nikuradse			**Haland**			**Churchill**			**Swamee**		
Θ	Δ_{SD}	$\Delta_{m,i}$	Θ	Δ_{SD}	$\Delta_{m,i}$	Θ	Δ_{SD}	$\Delta_{m,i}$	Θ	Δ_{SD}	$\Delta_{m,i}$
0.99647	0.0090	-0.08551	0.99932	0.00262	-0.03665	0.99997	7.5682E-05	0.05515	0.9752487	0.003714	0.05517

equations like Nikuradse, Haland, Churchill and Swamee.

Elfman's (1993) and Marengo-Mogollón (2005) theoretical analysis was reviewed and was proposed two Scenario's analysis. They were compare with models measurements and they were validate. When comparing the results obtained from simple equations with those determined from the experiments their validity is demonstrated to calculate coefficients λ_b, λ_w and K_w (Scenario I), and λ_b, λ_w, λ_C, and $K_{mC,i}$ in Scenario II. The theoretical development is quite acceptable.

Theoretical model can be applied in model development with this kind of analysis.

In future research, will be possible to analyze the hydraulic behavior in tunnels having other geometries (horseshoe and circular cross sections with composite roughness), so as to be able to define the behavior of the tunnels when functioning as open channels.

It will be possible to validate the equations proposed for the composite roughness with their respective geometries, and to investigate the behavior of the equal-velocity curves with tunnels operating as full pipes.

It should be made special mention in order to measure in prototype tunnels and compare with the results obtained in hydraulic models.

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With respect to the statistical analysis, standard deviation (Δ_{SD}), correlation ratio (Θ), and maximal relative errors ($maxRE^{+}$), are quite low.

The minimum values of standard deviation Δ_{SD} = 7.58 belongs to Churchill criteria besides the best correlation ratio Θ = 0.9999682. The maximal relative error belongs to Swamee $maxRE^{+}$ = 0.0936 and the maximum negative error belongs to the Nikuradse criteria $maxRE^{-}$ = 0.144.

The minimum difference in the mean value $\Delta_m = \frac{K_{mC,i} - K_{pred,i}}{K_{meas}}$ is for Haland criteria (-0.03665) although the obtained for Churchill (0.05517) and Swamee (0.05515) are very similar.

Under this analysis it is possible to say that the best theoretical criteria in order to obtain an accurate behavior is with Churchill criteria that offers the best correlation ratio, followed by Haland, Nikuradse and Swamee.

If it is selected standard, division also is Churchill criteria. If mean criteria is selected, Haland gives the best approximation.

The results obtained with Haland criteria are very good also.

Notation

A = cross sectional area.

a = height of triangle defining the area of influence of the floor material in tunnels with composite roughness.

A_w and A_w = areas of influence of the material for the floor slab and for walls and vault, respectively.

d = depth of water at tunnel entrance.

D = pipe diameter.

d/D = ratio between inlet hydraulic head with respect to the tunnel equivalent diameter.

D_e = equivalent diameter to the circular cross section.

f = Marchi's shape factor.

F = total shear force.

F_b and F_w = shear forces at floor, and at walls and vault, respectively.

g = acceleration of gravity.

h = lost of energy.

K = absolute roughness of the material of the circular conduit.

K_b and K_w = roughness coefficient at floor and at walls and vault, respectively, in a tunnel with composite roughness.

K_n = conversion factor *of units for* Manning's formula.

K_s = equivalent absolute roughness of the material for non-circular conduits.

l = length of the section.

L_e = scale of lines.

LP = area of section under study.

n_e = scale of roughness.

P = perimeter.

Q = flow rate.

Q_e = scale of flow rates.

Re = Reynolds' number.

Rh = hydraulic radius.

R_R = Reynolds' number calculated with the hydraulic radius.

S = hydraulic gradient.

S_1 *and* S_2 = slopes of tunnels.

V = mean velocity.

V_b and V_w = mean values at floor, walls and vault.

V_e = scale of velocities.

V_{max} = maximum flow velocity.

V_{real} = real velocity registered with a Prndtl-Pitot tube.

ρ = density of fluid.

Δ = increment as a percentage.

υ = kinematic viscosity of water.

τ = shear stress at the wall.

λ, n, C = coefficients of resistance to flow by Darcy-Weisbach, Manning and Chezy, respectively.

τ_b and τ_w = shear stresses at perimeter of floor slab and at perimeter of walls and vault, respectively.

λ_b and λ_w = coefficients of resistance to flow at floor, and at walls and vault, respectively.

λ_c = coefficient of resistance calculated with programs developed by Marengo-Mogollón.

Δh_b and Δh_w= hydraulic head losses at floor and at walls and vault, respectively;

λ_m = average coefficient of resistance.

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Annex I. Roughness criteria for vault section.

Table I. Values of the dimensionless coefficient λ and absolute roughness to estimate head losses in pressurized conduits.

Author	Value of λ		Value of K	Eq.
Nikuradse (1933)	$\frac{1}{\sqrt{\lambda}} = 2\log\frac{3.71D}{K}$ Pipes $K=\frac{2A/P}{10^{1/2}\left(\lambda^{-1/2}-1{,}74\right)}$ $\frac{1}{\sqrt{\lambda}} = 1{,}74 + 2\log\frac{2A}{KP}$ Other shape	I.1	$K = \frac{2A/P}{10^{1/2\left(\lambda^{-1/2}-1{,}74\right)}}$	I.1I.1I.1I A.1
Haaland (1983) (Franzini & Finnemore, 1999)	$\lambda = \frac{1}{\left[-1.8\log\left(\frac{2K}{14{,}83b}\right)^{1.11} + \frac{6{,}90}{\left(\frac{2Vb}{\nu}\right)}\right]^2}$		$k = \left(\frac{14.83b}{2}\right)\left[\left(10^{-0.5555\sqrt{\lambda}} - \frac{6{,}90}{\left(\frac{2Vb}{\nu}\right)}\right)^{0.9009}\right]^2$	A.2
Churchill-Barr (Bombardelli & García, 2003)	$\lambda = \frac{1}{4\left[-\log\frac{2K}{14{,}83b} + \frac{5{,}76}{4\left(\frac{2Vb}{\nu}\right)^{0{,}9}}\right]^2}$		$K = \left(\frac{14{,}83b}{2}\right)\left[10^{-1/2\sqrt{\lambda}} - \frac{5.76}{4\left(\frac{2Vb}{\nu}\right)^{0.9}}\right]$	A.3
Swamee-Jain (1976) (Bombardelli & García, 2003)	$\lambda = \frac{0.25}{\left(-\log\frac{2K}{14{,}83b} + \frac{5{,}76}{\left(\frac{2Vb}{\nu}\right)^{0{,}9}}\right)^2}$		$K = \left(\frac{14{,}83b}{2}\right)\left(10^{-1/2\sqrt{\lambda}} - \frac{5.76}{\left(\frac{2Vb}{\nu}\right)^{0.9}}\right)$	A.4
Colebrook (Franzini & Finnemore, 1999)	$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{2K_C}{14.83b} + \frac{2.52}{\frac{2Vb}{\nu}\sqrt{\lambda}}\right)$		$K = \left(\frac{14.83b}{2}\right)1\left(0^{-1/2\sqrt{\lambda}} - \frac{2.52}{\frac{2Vb}{\nu}\sqrt{\lambda}}\right)$	A.5
Junke Guo-Pierre & Julien (IAHR, 2004)	$\lambda = \frac{0.3164}{R_e^{1/4}}\left(1 + \frac{R_e}{4.31\times10^5}\right)^{1/8}$			A.6

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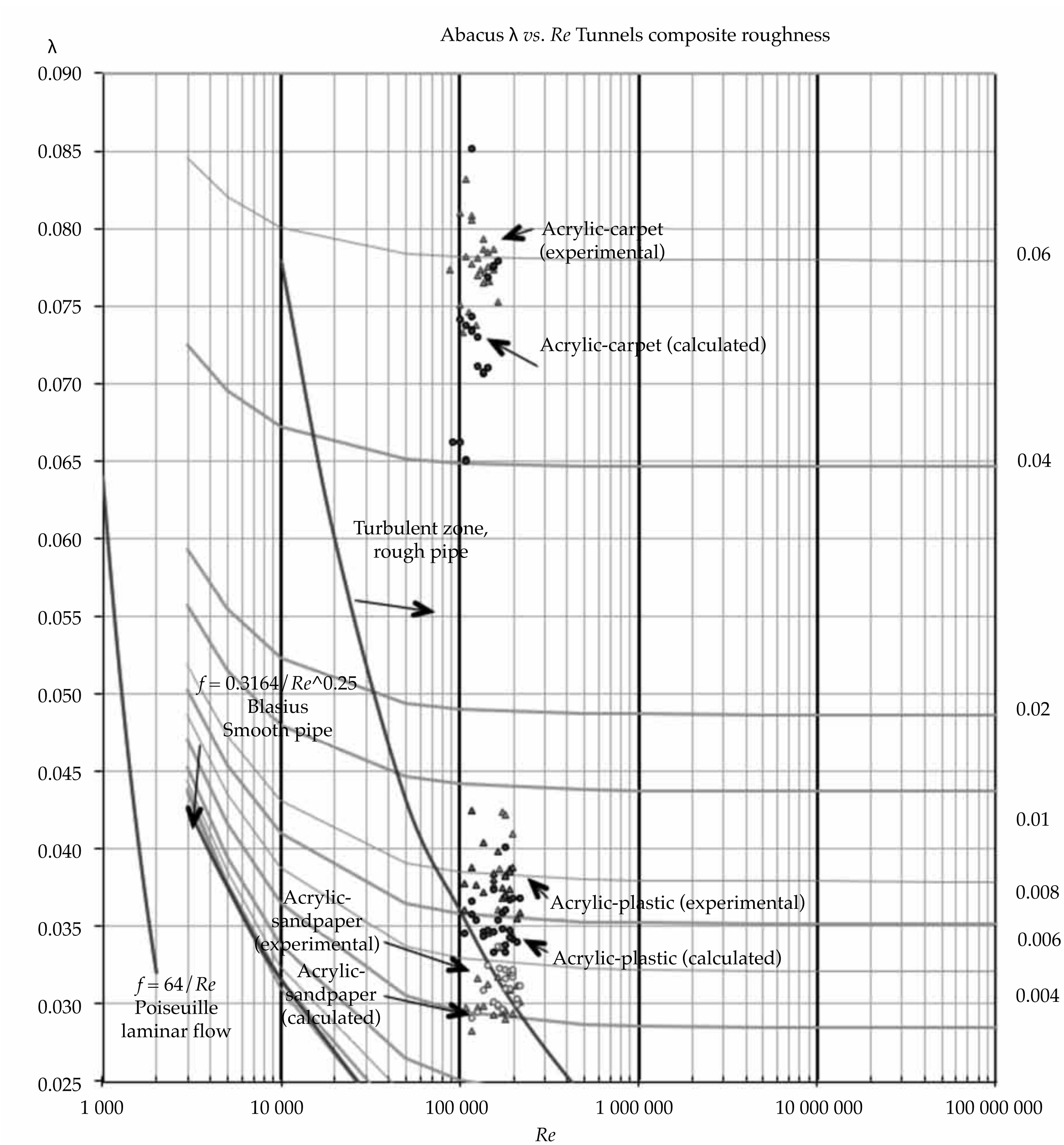

Figure 5. Abacus of λ *vs.* R_e curves for all tunnels with uniform and composite roughness.

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