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Articles

# A review of the methodologies for estimating the coefficient of losses in pipe curves under turbulent flow

# Una revisión de las metodologías para estimar el coeficiente de pérdidas en curvas de tuberías bajo flujo turbulento

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**Abstract**

A review of the methodologies reported in the literature is presented to estimate the coefficient of losses in simple pipe curves and installed in series with L-shape, S, U, and Z. The information is presented organized in chronological order and homogenized for comparison, providing the coefficient according to the angle of the curve, the ratio of the radius to the internal diameter and the Reynolds number. From this analysis, a summary is displayed where characteristics and parameters considered in each methodology are contrasted, as well as the presentation of results and validity specifications. With this it has been found that different methodologies are available to obtain the loss coefficient, through which different values are obtained, which may be due mainly to inequalities in the material analyzed and, in the factors, adopted in the investigations. The most studied simple and installed series curves are of 90º angle with gradual condition and U-shape (180º), respectively. The most suggested works to estimate the coefficient in curves are those of Kirchbach (1929) and Ito (1959, 1960), however, the Idel'chik (1966) manual presents the most extensive information for all the devices studied.

**Keywords**: Pipe curves, loss of energy, loss coefficient, turbulent flow, flow in pipes, hydraulic modeling.

## Resumen

Se presenta una revisión de las metodologías reportadas en la literatura para estimar el coeficiente de pérdidas en curvas simples de tuberías e instaladas en serie con forma en L, S, U y Z. La información se expone organizada en orden cronológico y homogenizada para su comparación, proporcionando el coeficiente en función del ángulo de la curva, de la relación del radio con respecto al diámetro interno y del número de Reynolds. A partir de este análisis, se exhibe un sumario, donde se contrastan características y parámetros considerados en cada metodología, así como la presentación de resultados y especificaciones de validez. Con ello, se ha encontrado que se dispone de metodologías diferentes para obtener el coeficiente de pérdidas, mediante las cuales se obtienen valores distintos, que pueden deberse principalmente a desigualdades en el material analizado y en los factores adoptados en las investigaciones. Las curvas simples e instaladas en serie más estudiadas son de ángulo de 90º con condición gradual y en forma-U (180º), respectivamente. Los trabajos más sugeridos para estimar el coeficiente en curvas son los de Kirchbach (1929) e Ito (1959, 1960); sin embargo, en el manual de Idel'chik (1966) se presenta la más amplia información para todos los dispositivos estudiados.

**Palabras clave**: curvas de tuberías, pérdida de energía, coeficiente de pérdida, flujo turbulento, flujo en tuberías, modelación hidráulica.

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# Introduction

A challenge that human beings have always had is the efficient and safe use of water resource at the lowest possible cost. For this, it is necessary to exploit and control hydraulic energy, that is, the potential energy and the kinetic energy that the fluid acquires (Nasir, 2014; Yuce & Muratoglu, 2015), establishing a direct relationship with the second law of thermodynamics (Herwig & Wenterodt, 2011), where it is essential to quantify the unusable energy (loss) to know the available hydraulic energy and use it in the systems (Schmandt, Iyer, & Herwig, 2014), such as: the pipelines in hydroelectric plants (Elbatran, Yaakob, Ahmed, & Shabara, 2015), the water distribution networks in populations (Yildirim & Singh, 2010), the irrigation systems (Sesma, Molina-Martínez, Cavas-Martínez, & Fernández-Pacheco, 2015), the pipelines in industrial plants (Anaya-Durand, Cauich-Segovia, Funabazama-Bárcenas, & Gracia-Medrano-Bravo, 2014), the distribution networks in geothermal power plants (Maria-Di, 2000), among others. However, there is the same interest in knowing the energy available in pipelines with fluids other than water, which are used in multiple piping systems of industrial plants (Perumal & Ganesan, 2016). This interest that occurs to evaluate the energy losses in the fluid, is due to the fact that they mainly affect the selection of the pipe and its diameter, as well as the pumping equipment. (Yoo & Singh, 2010).

Currently, the study of energy losses is divided into friction losses and local losses (Liu, Xue, & Fan 2013), both of different origin, but with equal importance. The research focuses on the latter and specifically in the direction change devices known as "curves", since they are widely used in piping systems to adjust the direction of the ducts (Gontsov, Marinova, & Tananaev, 1984). However, the other changes of direction called "elbows" are also widely used, but they deserve a separate review. (Kast, 2010; Zmrhal & Schwarzer, 2009). Finally, since pipe curves are widely used in hydraulic systems, various investigations have been carried out on the energy losses they generate, which have not stopped in recent years, as can be seen in the works of: Dang, Yang, Yang and Ishii (2018); Daneshfaraz, Rezazadehjoudi and Abraham (2018); Reghunathan, Son, Suryan and Kim (2019); Friman and Levy (2019); Tripathi, Portnikov, Levy and Kalman (2019); Du *et al*. (2020); Arun, Kumaresh, Natarajan and Kulasekharan (2020); Jia and Yan (2020). However, these recent investigations are based on results from computational models, which are generally validated against experimental results from physical models, the latter being the ones that will be analyzed in this study.

Investigations of energy losses in pipe curves, as well as in other devices, focus particularly on estimating the loss coefficient ($\xi$), however, despite numerous efforts in different scenarios, it has not been satisfactorily agreed on the results of this coefficient (Dutta & Nandi, 2015; Ito, 1960; Kilkovsky, Jegla, & Stehlik, 2011). This leads to uncertainty in the choice of data and methodology to be used, adding that generally manuals and textbooks do not indicate the considerations adopted in the studies and for which types of pipes they are applicable. Due to such relevance, the objective of this work is to present a review of the investigations and methodologies reported in the literature to estimate the coefficient $\xi$ in pipes curves under turbulent flow, that makes it possible to visualize important details of the studies and a contrast of

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their characteristics and scopes, from organized and homogenized information.

## Nature and evaluation of energy losses

Pipes curves are used in sudden and gradual condition (Figure 1). When a fluid circulates through a straight conduit of constant cross section and crosses these devices, energy losses occur in the environment where they are located (Bariviera, Frizzone, & Rettore, 2014; Fuentes & Rosales, 2004). This is a consequence of the modification of the direction of the flow that alters the distribution of speeds and pressures, causing the fluid to separate in the internal part of the curve and collide on the external part, which has repercussions in vortices and an increase in the pressure; this implies that after the curve the movement in the flow is developed in a spiral, up to lengths approximately 50 times the diameter of the duct (Chowdhury, Biswas, Alam, & Islam, 2016; Ito, 1960; Villegas-León *et al*., 2016). However, when the curve is presented gradually (Figure 1b), the flow disturbance phenomenon is similar to that caused by sudden change (Figure 1a), but it occurs to a lesser extent (Hager, 2010; Kast, 2010; Villegas-León *et al*., 2016).

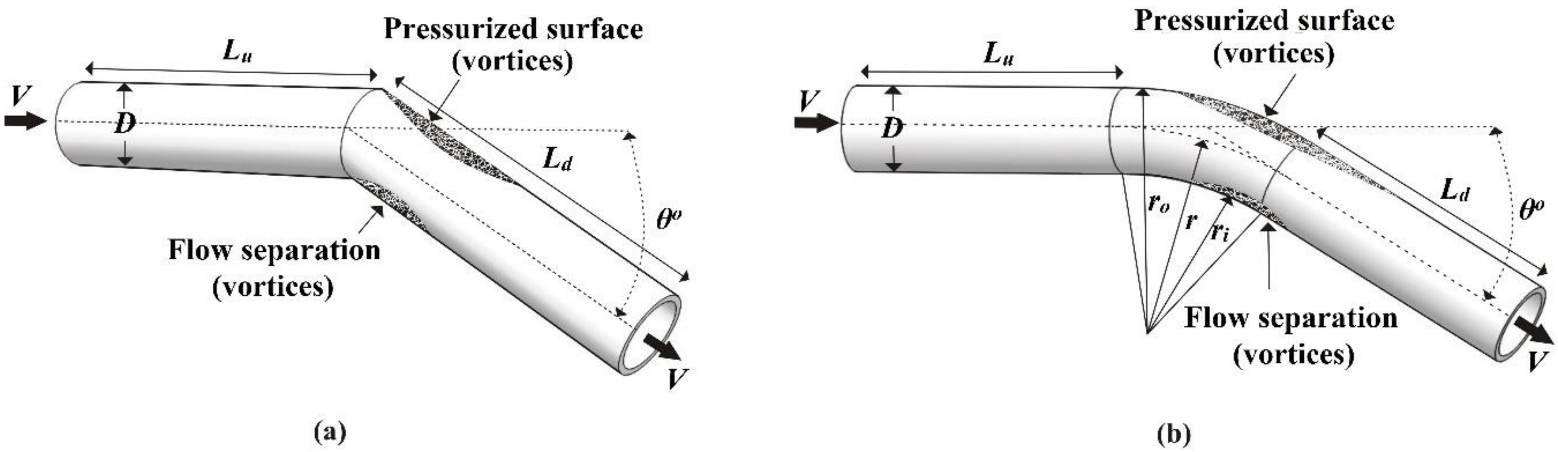


**Figure 1**. Behavior of the circulating flow in: a) Sudden curve. b) Gradual curve. $\theta$ is the angle of deflection of the curve; $D$ is the diameter of the duct; $V$ is the average flow velocity; $r$ is the radius of curvature with respect to the axis of the device; $r_i$ is the internal radius of curvature; $r_o$ is the external radius of curvature of the device; $L_u$ and $L_d$ are the lengths of the pipes upstream and downstream of the curve, respectively.

According to Beij (1938), Ito (1960), Idel'chik, (1966), Yildirim and Sing (2010), Sotelo (2013), Acero and Rodríguez (2008), USACE (1980), and USBR (1985), among others, local losses are determined by the following general equation:

$$h_L = \xi_c \frac{V^2}{2g} \tag{1}$$

where $h_L$ is the total hydraulic energy loss (m), which in this work will refer to those caused by pipe curves; $g$ is the acceleration of gravity (m/s$^2$); $V$ is the average velocity of circulation (m/s), which in a change of direction is the one that occurs upstream of the device; $V^2/2g$ is the charge for velocity or kinetic energy (m); and $\xi_C$ is the loss coefficient of the curve (dimensionless), which is determined experimentally. To define the value of the loss coefficient of a sudden curve ($\xi_{SC}$), generally only the angle $\theta$ is required, while for the loss coefficient of a gradual curve ($\xi_{GC}$), the angle $\theta$ and the ratio of the radius of the curve with respect to the diameter of the duct ($r/D$) are demanded; however, sometimes both coefficients are related to the Reynolds number ($R_e$), which for circular ducts can be expressed as:

$$R_e = \frac{\rho V D}{\mu} \qquad (2)$$

where $R_e$ is dimensionless; $\rho$ is the density of the fluid (kg/m$^3$); $V$ is the average velocity of the flow (m/s); $D$ is the diameter of the duct (m); $\mu$ is the dynamic viscosity of the fluid (kg/m·s).

Now, it is said that the energy loss in curves is compounded as follows (Beij, 1938; Crane Co., 1982; Ito, 1960):

$$h_L = h_c + h_d + h_f \qquad (3)$$

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in which $h_c$ is the loss due to the alteration of the flow by the type of curve, which is obtained from the pressure head difference at the beginning and end of the device; $h_d$ is the excessive loss in the length of the downstream pipe, relative to the normal friction loss; $h_f$ is the friction loss in the length of the curve, which can be obtained using the Darcy-Weisbach equation which is written as follows:

$$h_f = f\frac{L}{D}\frac{V^2}{2g} \tag{4}$$

where $L$ is the length of the pipe (m); $f$ is a coefficient of friction that depends mainly on $D$, the roughness of the pipe ($\varepsilon$) and $R_e$; however, $f$ is widely studied and is provided in documents on the subject, such as USACE (1980), Crane Co. (1982), CFE (1983), USBR (1985), nd Saldarriaga (2016), among others.

On the other hand, it should be mentioned that from $h_L$ the friction loss in the length of the curve is frequently excluded to be treated as such, together with the losses caused by straight sections of pipes in a system. In this way, the total energy loss in a curve would be as:

$$h_L = h_c + h_d \tag{5}$$

Then, knowing the total energy loss, the loss coefficient of a curve is given by the following expression:

$$\xi_C = \frac{h_L}{V^2/_{2g}} \quad (6)$$

Finally, it is important to verify whether or not the friction loss in the curve path is considered in $h_L$, since this affects the value of the loss coefficient. The following section presents research results and methodologies reported in the literature to estimate the loss coefficient of pipe curves.

## Loss coefficient estimation

This section presents an analysis of the research and methodologies reviewed in the literature on energy losses and the loss coefficient of pipe curves, where five specific cases are analyzed: the case of a simple device (Figure 1) and four other cases of structures formed with simple devices in series (one followed by another), which are classified as L-shape, S-shape, U-shape and Z-shape (Figure 2). In the mentioned study scenarios, the pipe lengths will be treated as proportional to their internal diameters ($D$) or to the radius of the curve ($r$). Some cases in which

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authors only mention the nominal diameter of the pipes ($Dn$), the values of $D$ will be estimated from the values of $Dn$ indicated in Shames (1995), for carbon steel pipes, steel alloy pipes and stainless steel pipes.

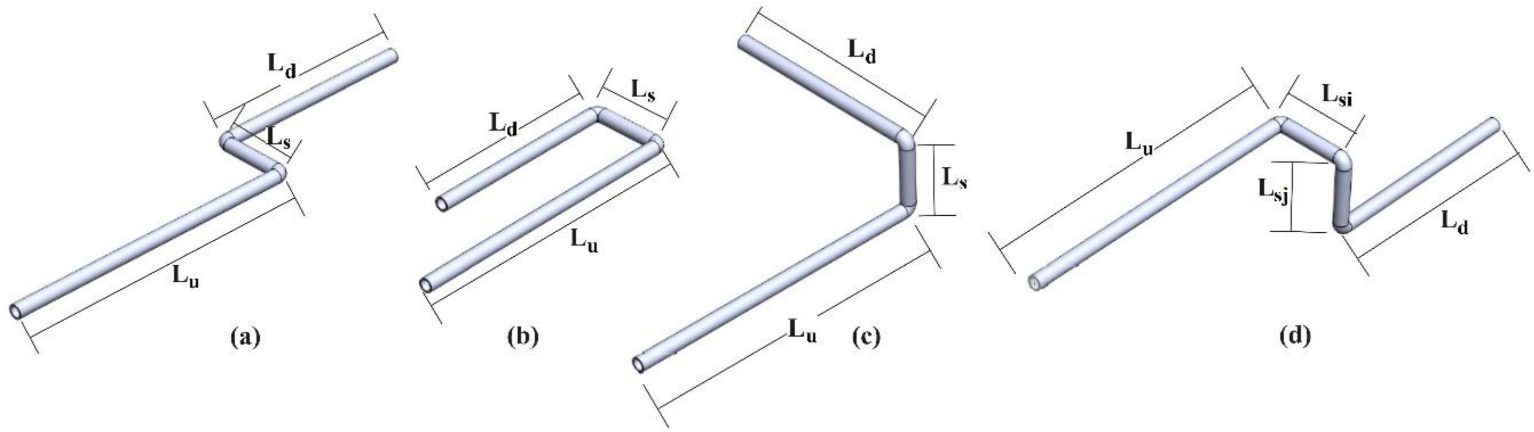


**Figure 2**. Study cases of structures with series curves: a) Z-shape, b) U-shape, c) L-shape and d) S-shape. $L_u$ and $L_d$ are the lengths of the pipe upstream and downstream of the device, respectively; $L_s$, $L_{si}$ and $L_{sj}$ are lengths of separation between two devices placed in series.

One of the widely cited works is Beij (1938), who carried out an experimental investigation of the pressure loss caused by gradual curves of steel with $\theta$ = 90º, $Dn$ = 10.16 cm and $D$ = 10.23 cm; however, $h_f$ values were also obtained at $L_u$ = 48.15$D$ and $L_d$ = 168.12$D$. For this, tests were carried out with water in 9 horizontal curves of 9.92 ≤ $r$ ≤ 204.2 (cm). Height differences pressure were evaluated in different parts of the system through a series of piezometers, both upstream and downstream of the device, while the velocity was estimated from the volumetric flow rate and the area of the cross section of the duct. With this, results of the coefficient of the gradual curves ($\xi_{GC}$) were obtained for relations of 0.97 ≤ $r/D$ ≤ 19.96 and approximate values of 0.23x10$^5$ ≤$R_e$≤ 3.4x10$^5$. The $\xi_{GC}$ values did not present a defined trend for $R_e$ values, therefore average values of $\xi_{GC}$ are also presented only in function of $r/D$. These results are presented in a graph, where it is shown that the coefficient decreases from 0.37 to 0.16 and then increases from 0.16 to 0.41, when the $r/D$ ratio increases from 1 to 3.5 and from 3.5 to 20, respectively; therefore, the minimum and maximum value of $\xi_{GC}$ is 0.16 and 0.41, when $r$ is 3.5 and 40 cm, respectively. These values were compared in another graph to the data obtained experimentally by: Hofmann (1929), in smooth and rough curves of $D$ = 4.32 cm; Davis (1911), in curves of $D$ = 5.08 cm; Balch (1913), in curves of $D$ = 7.62 cm; Vogel (1933), in curves of $D$ = 15.24, 20.32 and 25.40 cm; and by Brightmore (1907), in curves of $D$ = 7.62 and 10.16 cm. In this comparison of results, the values of $\xi_{GC}$ are not congruent between one author and another. However, the maximum value of $\xi_{GC}$ = 0.58 and is given by Balch (1913) with the relation $r/D$ = 15, while the minimum value of $\xi_{GC}$ = 0.08 and is obtained from Hofmann (1929) in smooth pipes, with the relation $r/D$ = 7.

In order to evaluate the behavior of the energy loss in the gradual curves of cast brass, Ito (1960) carried out experiments in devices with approximately values of $D$ = 3.5 cm and of $\theta$ = 45, 90 and 180º. The 180º curve is a U-shaped structure, which is formed by two series curves of

90º deflection, which produce an $L_s$ = $2r$. The 45º curves had values of $r$ = 6.42, 11.3 and 25.54 cm; the 90º curves presented values of $r$ = 3.52, 6.38 and 11.42 cm; and the 180º curves were with values of $r$ = 6.39 and 11.32 cm. Upstream and downstream copper pipes were coupled to the studied devices, with $L_u$ = 153.8$D$ and $L_d$ > 71.6$D$. For the practice, water at constant head was used, as well as a system of manometers to estimate the pressure heights at 5 points upstream of the curve and at 8 points downstream of it. At each point, the pressure heights were evaluated on the right and left side on the axis of the cross section, as well as in the upper and lower part of it, where $h_L = h_c + h_d$ was evaluated. Considering the above, values of $\xi_{GC}$ were obtained for a range of 0.2x10$^5$ ≤ $R_e$ ≤ 4.0x10$^5$, evaluating $h_d$ in values of $L_d$ = 0, 9$D$, 18$D$ and >50$D$; the results showed that the coefficient decreases as $R_e$ increases. Based on this, the author proposed equations to estimate the loss coefficient, which are also recommended in Lourenco and Xin (2017), which are set out below:

For $0.25R_e(r/D)^{-2} < 91$

$$\xi = 0.01746Kf\theta\left({r}/{D}\right) \quad (7)$$

For $0.25R_e(r/D)^{-2} > 91$

$$\xi = 0.004314 K\theta {R_e}^{-0.17} \left({r}/{D}\right)^{0.84} \tag{8}$$

where: $\xi$ is the loss coefficient of a gradual curve ($\xi_{GC}$) and of a U-shaped structure ($\xi_{US}$); $f$ is specified in Ito (1959); $K$ is a dimensionless coefficient that depends on $\theta$ and $r/D$, which is estimated by means of a graph or with equations suggested by the author, which can be written as follows:

If $\theta$ = 45º

$$K = 1 + 5.126\left({r}/{D}\right)^{-1.47} \tag{9}$$

If $\theta$ = 90º and 2($r/D$) < 19.7

$$K = 0.95 + 4.421\left({r}/{D}\right)^{-1.96} \tag{10}$$

If $\theta$ = 90º and 2($r/D$) > 19.7

$$K = 1.0 \tag{11}$$

If $\theta$ = 180º

$$K = 1 + 5.056\left({r}/{D}\right)^{-4.52} \tag{12}$$

It should be noted that, in White (2008) it is suggested to estimate the loss coefficient of a 90º curve, using the equation for $0.25R_e(r/D)^{-2}$ > 91. On the other hand, when comparing the results for the 45º, 90º and U-shape curves, a good correlation was shown between the experimental coefficients obtained and those estimated with equation (3), when $L_d$ > 50$D$. However, among various graphic presentations of the results, the following cases stand out:

a) The $\xi_{GC}$ results of the 45º curves are compared with the values estimated by equation (3), the Weisbach equation (Weisbach, 1855) and the Richter equation (Richter, 1930). The results of the Weisbach equation (Weisbach, 1855) adhered to those obtained experimentally with $L_d$ = 18$D$, while those determined by the Richter equation (Richter, 1930) were inclined to those investigated with $L_d$ = 0.

b) However, the values obtained for $\xi_{GC}$ of the 90º curves were compared against the values calculated with equation (3) and the equations proposed by: Weisbach (1855), Richter (1930), Pigott (1950) and Pigott (1957). In this part, the $\xi_{GC}$ values estimated with the Weisbach equation (Weisbach, 1855) were more congruent to those examined with $L_d$ > 50$D$; those determined with the Pigott equation (Pigott, 1957) were greater than all; and those obtained with the equations of Richter (1930) and Pigott (1950), were more inclined to the coefficients with low values $L_d$.

c) In the case of the U-shape curves, the same comparison criterion was used as in the 45º curves. Here, the $\xi_{US}$ values estimated using the Weisbach equation (Weisbach, 1855) were higher than in all cases, while

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those determined using the Richter equation (Richter, 1930) were more biased on low $L_d$.

d) On the other hand, another graph shows the experimental results of the loss coefficient in smooth curves with $R_e$ = 2x10$^5$ and θ = 45º, 90º and 180º (U-shape), as a function of $r/D$ from 1 to 15; the graph is also indicated in White (2008) to estimate the coefficient of losses in question. These data are compared with the values obtained by Equations (3), the one proposed by Hofmann (1929) and the one suggested by Wasielewski (1932). The author's experimental and analytical results showed good correlation with those given by Wasielewski (1932), when $\theta$ = 45º, and also with the results estimated by Hofmann (1929), when $\theta$ = 90º.

e) Finally, experimental results of the loss coefficient are exposed, which were studied in 10 copper curves with long radius, with $\theta$ = 45, 90, 135 and 180º (U-shape), as well as with $r/D$ from 5.2 to 108 ; it is highlighted that they have been previously reported in Ito (1956). These results indicated that, as the relation $r/D$ increases for the same value of $\theta$, $h_c$ also increases and $h_d$ decreases. In a graph is displayed the results of the coefficients $\xi_{GC}$ and $\xi_{US}$ for a range of 0.1x10$^5$ ≤ $R_e$ ≤ 3.0x10$^5$, where it is shown that said coefficients decrease as $R_e$ rises. Furthermore, it is illustrated that the minimum values of the coefficients are obtained with $\theta$ = 90º and $r/D$ = 7.6; however, the maximum values are given with $\theta$ = 45º and $r/D$ = 108.

Regarding commercial PVC curves, Chen-Tzu (1969) carried out an experimental study on the behavior of flow and energy losses in curves with $\theta$ = 90º and $r/D$ = 1.59. The devices used were 2 inches with $D$ =

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5.25 cm, which were coupled to pipes with the same characteristics and lengths $L_u$ = 100$D$ and $L_d$ > 36$D$. The tests were carried out in horizontal and vertical curves, using different water flows with a temperature of approximately 21.1 ºC, which produced $R_e$ from 0.282x10$^5$ to 1.86x10$^5$. By means of a system of piezometers, the pressures were evaluated at every 90º in the circumference of the cross-section, allowing to obtain the pressure heights on the left and right side of the pipeline axis and in the upper and lower part thereof; these pressures were estimated at the center of the curve angle and up to distances of 30$D$ upstream of the device and 36$D$ downstream. Among other results, a graph of the $\xi_{GC}$ is presented for the horizontal and vertical direction, according to the variation of $R_e$. In the graph shows that in both cases, the $\xi_{GC}$ decreases as the $R_e$ increases, but with different behavior. In the ranges approximated to $R_e$ < 0.57x10$^5$ and $R_e$ > 1.4x10$^5$, the $\xi_{GC}$ was slightly higher in the vertical curve, while the $\xi_{GC}$ was slightly higher in the horizontal case for the range of 0.57x10$^5$ < $R_e$ < 1.4x10$^5$; therefore, the $\xi_{GC}$ is the same for the horizontal and vertical curve, when $R_e$ ≈ 0.57x10$^5$ and 1.4x10$^5$, presenting values of 0.24 and 1.8, respectively. Finally, the maximum values of $\xi_{GC}$ in the horizontal and vertical curve were 0.35 and 0.38, while the minimum values were 0.16 and 0.18, respectively, which were presented in the minimum and maximum values of $R_e$.

More than a decade later, Turian, Hsu and Selim (1983) present an investigation on the loss coefficient for some devices and valves of steel pipes, where flows with suspended solids were used. This research is said to have been previously carried out by Hsu (1981), in which values of the loss coefficient were obtained for sudden and gradual curves of steel with $D$ = 2.54 and 5.08 cm, using different glass bead concentrates from 0 al 50% with respect to weight. The sudden curves were 90º of deflection, while the gradual ones had 45, 90 and 180º (U-shape, $L_s$ = 0). The 45º and U-shape curves presented $r$ = 3.81 cm and 5.72 cm, in the smallest and largest $D$, respectively. However, the 90º gradual curves were 3.81, 11.43, 21.59 and 31.75 cm radius for the $D$ minor, as well as 5.72, 22.86, 43.18 and 63.50 cm radius for the $D$ major. The results are presented in graphs and tables, from which the following is highlighted:

a) The percentage of solid concentration does not significantly affect the value of the loss coefficient in all the curves studied, tending to average in an asymptotic behavior.

b) The loss coefficient for devices with $D$ = 2.54 cm, are presented as a function of the $R_e$ from 0.1 to 0.8x$10^5$, where the loss coefficient tends to be asymptotic from $R_e$ ≥ 0.25x$10^5$.

c) The results of the coefficient in the devices with $D$ = 5.08 cm, were indicated with $R_e$ from 0.2 to 1.7x$10^5$, in which it was shown that said coefficient follows an asymptotic behavior in $R_e$ ≥ 0.4x$10^5$.

d) In a table shows average values of the loss coefficient for devices with $D$ = 2.54 cm and 5.08 cm for flows with $R_e$ ≥ 0.25x$10^5$ and $R_e$ ≥ 0.4x$10^5$, respectively. The coefficients of the $D$ minor devices were 0.94 in the 45º curve; 1.63 on the sudden 90º curve; the minimum of 0.80 and the maximum of 1.27 in gradual curves of 90º with $r$ = 21.59 and 3.81 cm, respectively; and 0.90 in U-shape curves. However, these coefficients in the devices with $D$ greater were 0.73 in the 45º curve; 1.91 in the sudden 90º curve; the minimum of 0.60 and the maximum of 1.13 in gradual curves of 90º with $r$ = 22.86 and 43.18 cm, respectively; and 0.92 in U-shape curves.

In addition to showing experimental results, Turian *et al*. (1983) mentions that the coefficient of losses in curves can be obtained through the equations of Ito (1959) and Ito (1960) for $0.25R_e(r/D)^{-2} < 91$ and $0.25R_e(r/D)^{-2} > 91$ ( previously exposed), and also, using a table indicated in Crane Co. (1982). The experimental results are only contrasted against those of Crane Co. (1982), where notable differences were found for the two types of diameters; the coefficients obtained in the research were always higher, except for the U-shape ones, which were lower than those given in the literature. In this comparison of the devices with $D$ = 2.54 cm, the minimum and maximum differences of the coefficients were 0.25 in the U-shaped curve and 0.67 in the 90º curve with r = 11.43 cm, respectively; meanwhile, these differences in the devices with $D$ = 5.08 cm were 0.03 in the U-shaped curve and 0.77 in the sudden 90º curve.

On the other hand, Gontsov *et al*. (1984) carried out a laboratory investigation on the behavior of flow and energy loss in a horizontal curve with θ = 90º, using isothermal air flow. The model was built with 0.6 cm thick organic glass, with $D$ = 20.6 cm, $r/D$ = 1, $L_u$ = 35$D$ and $L_d$ = 25$D$. The pressures in the flow and on the walls were evaluated using differential manometers with alcohol, considering $R_e$ values from 0.9 to $4.0 \times 10^5$. The results are indicated by graphs, however, four equations are also shown to obtain the $\xi_{GC}$. The first is an equation that you attribute to D. Al ’tshul, which is written as follows:

$$\xi_{GC} = [0.2 + 0.001(100f)^8](r/_D)^{-0.5} \tag{13}$$

The second equation presented is the one reported by Ito (1960) and previously cited in Equation (8). The two remaining equations are obtained by the authors for smooth gradual curves, based on their own research and the results of Ito (1960). The third equation is suggested for 0.2x10$^5$ < $R_e$ < 4.0x10$^5$, which is explained below:

$$\xi_{GC} = 1.45{R_e}^{-0.15}(r/_D)^{-0.3} \tag{14}$$

The fourth equation is recommended for $R_e$ ≥ 4.0x10$^5$, where the $\xi_{GC}$ is independent of the flow regime. This equation is written as:

$$\xi_{GC} = 0.21(r/_D)^{-0.3} \tag{15}$$

Finally, it should be noted that the experimental values obtained from the loss coefficient presented a good correlation with the results obtained in Ito (1960), when the range of 1 ≤ $r/D$ ≤ 3 is found.

Based on the analysis of the experimental information reported in the literature, in Villegas-León *et al*. (2016) equations are proposed for

$\xi_{SC}$ and $\xi_{GC}$, which generate average values from the values obtained from the coefficient with the methodologies considered. In the case of sudden curves, the methodologies indicated in CFE (1983), Miller (1990), Sotelo (2013) and USACE (1980) were selected, on which the following expression was obtained:

$$\xi_{SC} = \frac{0.0031960558+0.0030444516\,(\theta)}{1-0.014390831\,(\theta)+0.00006719314\,(\theta)^2} \tag{16}$$

It is indicated that the previous equation is valid for values of $0 < \theta \leq 90^{o}$. On the other hand, the methodologies of CFE (1983), Miller (1990), Sotelo (2013), SARH (1984), USACE (1980) and USBR (1985) were used on gradual curves, from which the following equation was obtained:

$$\xi_{GC} = \frac{A+C\theta}{B} \tag{17}$$

where: $A$, $B$ and $C$ are coefficients that depend on the relation $r/D$ and are determined by the following equations:

$$A = -0.0573379 + 0.00496834\left({}^{r}/_{D}\right) - 0.00001716\left({}^{r}/_{D}\right)^3 + \frac{0.07867083}{\left({}^{r}/_{D}\right)^{0.5}} - \frac{0.066727}{e^{\left({}^{r}/_{D}\right)}} \tag{18}$$

$$B = 0.20495202 + 0.05446522\left({}^{r}/_{D}\right) - 0.08723377\left({}^{r}/_{D}\right)^{0.5} \ln\left({}^{r}/_{D}\right) - \frac{0.45002930 \ln\left({}^{r}/_{D}\right)}{\left({}^{r}/_{D}\right)} - \frac{0.25130468}{\left({}^{r}/_{D}\right)^{2}} \quad (19)$$

$$C = -0.01383436 - 0.01385106\left({}^{r}/_{D}\right) + 0.00051449\left({}^{r}/_{D}\right)^{2} + 0.04504019\ln\left({}^{r}/_{D}\right) + \frac{0.08991395}{e^{\left({}^{r}/_{D}\right)}} \quad (20)$$

It is established that these equations for the gradual condition are valid under the ranges of $1 \leq r/D \leq 10$ and $5 \leq \theta \leq 90$º.

Now, in the remaining paragraphs, various methods are presented to estimate the loss coefficient of pipe curves, which are suggested in manuals and books on the subject, based on the author's own experiences and/or on the results of other researchers, without showing details of the experimental developments.

In Gibson's document (Gibson, 1930), it is said that the first energy loss experiments in pipe curves were carried out by Weisbach (1855), using steel pipes with $D$ = 3.175 cm. From these experiments and others, he proposed the following equations for the sudden and gradual curves:

$$\xi_{SC} = 0.946 \sin^2 \frac{\theta}{2} + 2.05 \sin^4 \frac{\theta}{2} \quad (21)$$

$$\xi_{GC} = \left[0.000222 + 0.000276\left({}^{r}/_{D}\right)^{-3.5}\right] \theta \quad (22)$$

The first equation, which is for the sudden curve coefficient, is also suggested for elbows of the same condition. Additionally, tabulated values of $\xi_{GC}$ are presented, which are derived from Brightmore (1907) results for cast iron curves with $D$ = 7.62 and 10.16 cm, $r/D$ from 2 to 10 and velocities from 1.52 to 3.05 m/s. These results did not show a well-defined behavior before the variation of $r/D$ and the flow velocity, however, for the two diameters studied, the minimum and maximum values can be assumed to be presented at $r/D$ = 10 and 2, respectively. In the curves with $D$ = 7.62 cm, the lowest and highest coefficients were approximately 0.22 and 0.43, while for $D$ = 10.16 cm, they were 0.23 and 0.38, respectively. Finally, values of the loss coefficient in curves are also presented, estimated from Schoder (1908) research on wrought iron devices, which presented a $Dn$ = 6 inches, an estimated $D$ of 15.4 cm and $r/D$ ratios of 1.34 to 20. The values are shown in a table as a function of $r/D$ and three different speeds, 1.52, 3.05 and 4.88 m/s. In this case, the behavior of the coefficient was oscillatory, but taking into account the average of the speeds, it presented the minimum value of 0.27 in $r/D$ = 5 and 10, while the maximum value was 0.45 in $r/D$ = 1.34.

Now, Daugherty and Ingersoll (1954) establish that in gradual curves with θ = 90º, the loss coefficient is obtained by means of Pigott's equation (Pigott, 1950), which is written as follows:

$$\xi_{GC} = 0.106\left({}^{r}/_{D}\right)^{-2.5} + 2f^{2.5} \qquad (23)$$

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Since the value of $R_e$ affects $f$, it implies that it also influences $\xi_{GC}$. Finally, it is indicated that in curves with $\theta \neq 90º$, the value of $\xi_{GC}$ is approximately proportional to the value of $\theta$. That is to say, in a 45 and 180º curve, the value of $\xi_{GC}$ is 50 and 200%, respectively, over the value obtained with the previous equation.

In the Idel'chik manual (Idel'chik, 1966), extensive information on energy losses in the devices under study is presented, where it is established that the total loss is made up of the local loss and the friction loss along the curves. On friction losses, the author provides a series of equations, tables and graphs to obtain the coefficient $f$, however, it will not be abundant in this. Regarding local losses (equation 1), a range of equations, tables and graphs are presented, which are obtained from the experimental results of various authors. According to this information, the following cases were classified for simple curves and series curves:

a) If the curves are simple, gradual and with smooth surfaces ($\varepsilon \approx 0$), which comply with $0 < \theta \leq 180º$, $r/D \geq 0.5$ and $R_e \geq 2 \times 10^5$, the loss coefficient is obtained under the Abramovich equation (Abramovich, 1935):

$$\xi_{GC} = \delta\alpha \tag{24}$$

in which $\delta$ and $\alpha$ are coefficients that depend on $\theta$ and $r/D$, respectively, which are estimated from graphs and tables established by Nekrasov (1954), based on the investigations of Idel'chik (1953), Evdomikov (1940) and own. There the coefficient $\delta$ increases from 0 to 1.4, when $\theta$

increases from 0 to 180º, while $\alpha$ decreases from 1.18 to 0.17, when $r/D$ increases from 0.5 to 1.5, respectively. However, the following equations are also established to estimate the coefficients $\delta$ and $\alpha$:

If $\theta \leq 70º$:

$$\delta = 0.9 \sin \theta \tag{25}$$

If $\theta = 90º$:

$$\delta = 1.0 \tag{26}$$

If $\theta \geq 100º$:

$$\delta = 0.7 + 0.0039\theta \tag{27}$$

If $0.5 \leq r/D \leq 1.0$:

$$\alpha = 0.21\left({}^{r}/_{D}\right)^{-2.5} \tag{28}$$

If $r/D > 1.0$,

$$\alpha = 0.21\left({}^{r}/_{D}\right)^{-0.5} \tag{29}$$

b) When the curves are simple and gradual, with rough surfaces ($\varepsilon > 0$), short $r$, $0 < \theta \leq 180º$ and $0.5 < r/D \leq 1.5$, to drive flows with $R_e \geq 0.03x10^5$, the coefficient is estimated with the following equation:

$$\xi_{GC} = \delta\alpha\lambda_1\lambda_2 \tag{30}$$

where the coefficients $\delta$ and $\alpha$ are determined according to what is indicated in a); $\lambda_1$ and $\lambda_2$ are coefficients that are estimated by tables based on $R_e$, $r/D$ and $\varepsilon/D$. $\lambda_1$ and $\lambda_2$ can increase from $45f$ to 1.0 and 1.0 to 2.0, respectively, as the value of the parameters that govern them also increase.

c) If the curves are simple and gradual, with rough surfaces ($\varepsilon > 0$), long $r$, $0 < \theta \leq 180º$ and $1.5 < r/D \leq 50$, where flows with $R_e \geq 0.03x10^5$ are presented, the $\xi_{GC}$ is also determined with equation (30), but now the author provides new tables and graphs to estimate the coefficients $\delta$, $\alpha$, $\lambda_1$ and $\lambda_2$. There the coefficient $\delta$ can increase from 0 to 1.4, when $\theta$ increases from 0 to 180º, while $\alpha$ decreases from 0.21 to 0.30, when $r/D$ increases from 1 to 50, respectively. However, the coefficients $\lambda_1$ and $\lambda_2$ increase from $64f$ to 1.0 and from 1.0 to 2.0, respectively, as the value of $R_e$ and $\varepsilon/D$ also increases.

d) In the case of simple and gradual curves with smooth surfaces ($\varepsilon \approx 0$), long $r$, $0 < \theta \leq 90º$ and $r/D \gg 1.5$, to drive flows with $50 < R_e < 0.2x10^5$, the local loss coefficient includes the coefficient resistance to the total friction of the curve, therefore it should not be evaluated separately. In this case, the $\xi_{GC}$ must be estimated under the Aronov (1950) equation, which is based on the results of Adler (1934), White (1929) and own; this equation is written as follows:

$$\xi_{GC} = 0.0175 f \left( {}^{r}/_{D} \right) \theta \quad (31)$$

where $f$ is obtained through tables and graphs as a function of $r/D$ and $R_e$. In such data, the value of $f$ decreases as its governing parameters increase. The maximum value of $f$ is 0.34, when $r/D$ = 3.1 and $R_e$ = 400, while its minimum value is 0.029, when $r/D$ = 100 and $R_e$ = 0.2x10$^5$. On the other hand, three equations are also provided to estimate $f$, which were obtained by Aronov (1950), from the results of the aforementioned investigations which when substituting them in the previous equation, the following occurs:

If 50< $R_e(D/2r)^{0.5}$ ≤600:

$$\xi_{GC} = 0.31 \theta R_e^{-0.65} \left( {}^{r}/_{D} \right)^{0.825} \quad (32)$$

If 600< $R_e(D/2r)^{0.5}$ ≤1 400:

$$\xi_{GC} = 0.1557 \theta R_e^{-0.55} \left( {}^{r}/_{D} \right)^{0.775} \quad (33)$$

If 1400< $R_e(D/2r)^{0.5}$ ≤5000,

$$\xi_{GC} = 0.0723\theta R_e{}^{-0.45}\left({}^{r}/_{D}\right)^{0.725} \tag{34}$$

e) On the other hand, a graph is presented to estimate the coefficient of losses in simple curves of sudden condition and with 90º of deflection, which is applicable both for curves with $r_i$ = 0 and $r_o$ ≥ 0, and in curves with $r_i$ ≥ 0 and $r_o$ = 0. In the case that $r_o$ = 0, the $\xi_{SC}$ is greater than when $r_i$ = 0. The approximate minimum and maximum values that this coefficient can take with $r_o$ = 0 are 1.1 and 1.38, when $r_i/D$ is 1.1 and 2.1, respectively; while with $r_i$ = 0 it can take the values of 0.18 and 1.1, when $r_o/D$ is 1.2 and 0, respectively.

f) When there is a simple curve or elbow, with a sudden condition ($r/D$ = 0), smooth surfaces ($\varepsilon$ ≈ 0) and 0< $\theta$ ≤ 180º, which conducts flows with $R_e$ ≥ 0.4x10$^5$, the loss coefficient must be determined by Abramovich's equation (Abramovich, 1935), which is written as follows:

$$\xi_{SC} = \delta_1\delta_2 \tag{35}$$

where $\delta_1$ and $\delta_2$ are coefficients that depend on $\theta$. $\delta_1$ is obtained from tables and graphs with data from Richter (1930), Richter (1936) and Schubart (1929), where its value decreases from 3.0 to 1.2, when $\theta$ increases from 10 to 90º, respectively, while when 90 < $\theta$ ≤ 180º, the value $\delta_1$ remains constant at 1.2. However, the value of $\delta_2$ is obtained from tables and graphs with data from Weisbach (1855) or, by means of an equation proposed by this author, which, when replaced in the previous equation, adopts the following structure:

$$\xi_{SC} = \delta_1 \left(0.95 \sin^2 \frac{\theta}{2} + 2.05 \sin^4 \frac{\theta}{2}\right) \tag{36}$$

g) If the curve or elbow are simple, with a sudden condition ($r/D$ = 0), with rough surfaces ($\varepsilon$ > 0) and 0 < $\theta$ ≤ 180º, which conducts flows with $R_e$ ≥ 300, the coefficient must be estimated by the following equation:

$$\xi_{SC} = \delta_1 \delta_2 \eta_1 \eta_2 \tag{37}$$

in which $\delta_1$ and $\delta_2$ are determined according to what is indicated in f); $\eta_1$ and $\eta_2$ are coefficients that are determined in tables based on the value of $\varepsilon/D$ and $R_e$. The coefficient $\eta_1$ can increase from 45$f$ to 1.0, when the value of $R_e$ increases, while $\eta_2$ can increase from 1.0 to 1.5, when the values of $\varepsilon/D$ and $R_e$ increase.

h) If two series curves of gradual condition are presented, with smooth surfaces ($\varepsilon$ ≈ 0) and $r/D$ ≥ 0.5, forming a structure in the form of L, U or Z, to conduct flows with $R_e$ ≥ 2 x $10^5$, the coefficient of losses is determined by the following equation:

$$\xi_{GC} = K \left[\delta\alpha + \left(0.2 \frac{L_s - 2r}{D} + 0.0007 \frac{r}{D} \theta\right)\right] \tag{38}$$

where the coefficients $\delta$ and $\alpha$ are determined according to what is indicated in a); $K$ is a coefficient that depends on the relationship $(L_s - 2r)/D$, which is estimated using tables obtained from the results of Haase (1953). In the case of the L and Z shapes, the coefficient $K$ decreases from 2.5 to 2.0 and from 3.0 to 2.0, respectively, when the relationship $(L_s - 2r)/D$ increases from 0 to ≥1.0. However, in the case of the U-shape, this coefficient increases from 1.4 to 2.0, when the relation $(L_s - 2r)/D$ increases from 0 to ≥ 1.0.

i) Finally, when two gradual curves are presented in series, with rough surfaces ($\varepsilon$ > 0) and $r/D$ ≥ 0.5, establishing a structure in the form of L, U or Z, to conduct flows with $R_e$ ≥ 0.03x10$^5$, the coefficient of losses is determined by the following equation:

$$\xi_{GC} = K\left[\delta\alpha\lambda_1\lambda_2 + \left(0.2\frac{L_s - 2r}{D} + 0.0007\frac{r}{D}\theta\right)\right] \tag{39}$$

in which the coefficients $\delta$, $\alpha$, $\lambda_1$ and $\lambda_2$, are obtained as indicated in part b) and c) for relations 0.5< $r/D$ ≤1.5 and 1.5< $r/D$ ≤50, respectively; and the coefficient $K$ is estimated according to what is established in section h).

On the other hand, King, Wisler and Woodburn (1980) recommend tabulated values to estimate the $\xi_{GC}$, with $\theta$ = 90º and 1 ≤ $r/D$ ≤ 20, which are obtained from the results of Beij (1938); these results are shown graphically in Brater, King, Lindell and Wei (1996), and it is also indicated that in turbulent flow the value of $R_e$ does not significantly affect the $\xi_{GC}$. In these data the value of $\xi_{GC}$ decreases from 0.35 to 0.16, when $r/D$ increases from 1 to 4, while later it increases from 0.16 to 0.42, when $r/D$ takes values from 4 to 20, respectively. This indicates that the minimum value of $\xi_{GC}$ = 0.16 is originated at $r/D$ = 4, while the maximum value of 0.42 is presented at $r/D$ = 20. Finally, it is established in a very general way that, in gradual curves with $\theta$ = 45 and 180º, the value of $\xi_{GC}$ can be determined by means of 50% and 125%, respectively, of the values given for the curve with $\theta$ = 90º.

The USACE manual (USACE, 1980) presents what was established by Anderson (1947) to estimate the coefficient of losses in gradual and sudden curves, where the following is framed:

a) Regarding gradual devices, it is argued that Hofmann (1929) and Wasielewski (1932) established that the $\xi_{GC}$ is independent for $R_e$ > 2x10$^5$. Under this context, a graph with results from Wasielewski (1932) is recommended to estimate the $\xi_{GC}$ with smooth surfaces, $R_e$ = 2.25x10$^5$, 1 ≤ $r/D$ ≤ 10 and values of 0 < $\theta$ ≤ 90º; this graph is also suggested by Sotelo (2013). There, in general, the coefficient increases as the value of $\theta$ increases, but decreases as the ratio $r/D$ increases. However, the coefficient increases constantly and takes the same value in any relation $r/D$ for the angles $\theta$ from 0 to 25º, while if $r/D$ takes values of 4, 6 and 10, the coefficient decreases as $\theta$ = 65, 70 and 50º, respectively. It is also illustrated that the maximum value of $\xi_{GC}$ = 0.20 and occurs at $r/D$ = 1 and $\theta$ = 70º, while the minimum value is zero for any value of $r/D$, when $\theta$ = 0º.

b) In addition, the same graph presents fit curves on the results of Wasielewski (1932), which USACE (1980) recommends to obtain the $\xi_{GC}$ as a function of 1.5 ≤ $r/D$ ≤ 10 and 0 < $\theta$ ≤ 90º. According to this criterion, the $\xi_{GC}$ does not show fluctuations and increases as the angle $\theta$ increases, while it decreases with increasing $r/D$. Also, the minimum values are zero when $\theta$ = 0º, while the maximum is 0.16 when $r/D$ = 1.5 and $\theta$ = 90º.

c) Finally, if the devices are sudden, USACE (1980) and Sotelo (2013) also recommend a graph with results from Kirchbach (1929) and Schubart (1929) to estimate the $\xi_{SC}$, based on the ranges 0.2 ≤ $R_e$ ≤ 2.5x10$^5$ and 0 ≤ $\theta$ ≤ 90º. In this graph, the coefficient increases as $\theta$ increases, while it decreases when $R_e$ increases. The minimum and maximum values of $\xi_{SC}$ are presented at $\theta$ = 0º and 90º, espectively, for any value of $r/D$; the minimum values are zero, while the maximum values are 1.12.

Another document that is cited in the literature to determine the coefficient of losses in pipe curves is the Crane Co. (1982) manual, where the Pigott (1950) values are recommended to obtain the $\xi_{GC}$ of steel pipes, with $\theta$ = 90º, $D$ approximately 2.0 to 25.4 cm and $r/D$ = 1, 2 and 3. These data reveal that the coefficient decreases in relation to the increase in $D$, taking approximate values from 3.8 to 1.8, from 3.3 to 1.1 and from 3.5 to 1.25, when $r/D$ is equal to 1, 2 and 3, respectively, which indicates that the maximum values are obtained with $r/D$ = 1, while minimum values with $r/D$ = 2. In addition, the results of Beij (1938) for the $\xi_{GC}$, are also presented, which were discussed in USACE (1980). However, if the 90º curves are steel and flanged or with butt weld ends, a table is recommended to estimate the $\xi_{GC}$, based on the $r/D$ ratio of 1 to 20. In this table, the coefficient decreases from $20f$ to $12f$ and increases to $50f$, when $r/D$ takes values of 1, 2-3 and 20, respectively, presenting the minimum values in $r/D$ from 2 to 3. Also, it is specified that for a curve with $\theta$ = 180° (U-shape, $L_s$ = 0), the $\xi_{GC}$ = $50f$. Finally, a table with results from Kirchbach (1929) is recommended to estimate the coefficient in sudden curves of steel with $\theta$ from 0 to 90º, where the $\xi_{SC}$ increases from $2f$ to $60f$, respectively.

In SARH (1984) a table is suggested to estimate the coefficient of losses of gradual curves with $\theta$ = 90º, where the $\xi_{GC}$ is governed by the relation $r/D$ in the range from 1 to 10. In this table the $\xi_{GC}$ decreases from 0.52 to 0.18, when $r/D$ increases from 1 to 6, while later it increases from 0.18 to 0.20 as $r/D$ increases from 6 to 10. However, if the curve is with $\theta \neq$ 90º, the value of the $\xi_{GC}$ obtained will be proportional to a factor, which is estimated from a second table as a function of $\theta$; this factor increases from 0.20 to 1.30 according to the increase of $\theta$ from 10 to 180º, respectively. Lastly, it is commonly said, if the curve is sudden, the $\xi_{SC}$ can take values from 0.7 to 1.0, depending on the magnitude of $\theta$.

On the other hand, Pashkov and Dolqachev (1985) establish that in gradual curves with $D \leq$ 3.0 cm, the loss coefficient can be obtained from a table only as a function of $\theta$, where the value of $\xi_{GC}$ increases from 0.20 to 1.10, as $\theta$ increases from 30 to 90º. However, if the curve is sudden, the coefficient is obtained by the following expression:

$$\xi_{SC} = \xi_{90}(1 - \cos\theta) \quad (40)$$

where $\xi_{90}$ is the loss coefficient of a sudden 90º curve, which is estimated from a table as a function of $D$, in which $\xi_{90}$ decreases from 1.70 to 0.83, according to the increase in $D$ from 2.0 to 4.9 cm, respectively.

In the design of small dams, USBR (1985) recommends the graph previously discussed in Beij (1938) to determine the loss coefficient of a 90º gradual curve. However, in this graph this author includes an adjustment curve to obtain the $\xi_{GC}$ according to the relation $r/D$. There the coefficient decreases from 0.25 to 0.07 when $r/D$ increases from 1 to 4, respectively, then it remains constant at 0.07 when $r/D$ goes from 4 to 10. On the other hand, if the curve presents a $\theta \neq 90º$, corrects the value obtained from $\xi_{GC}$ in proportion to a factor obtained from a graph as a function of $\theta$. This factor increases from 0 to 1.12, when $\theta$ increases from 0 to 120º, respectively.

On the other hand, Simon (1986) suggests an equation to estimate the coefficient of losses of gradual curves, which can be written as follows:

$$\xi_{GC} = \left[0.13 + \frac{0.1635}{\left({r}/{D}\right)^{3.5}}\right]\left(\frac{\theta}{180}\right)^{0.5} \quad (41)$$

Additionally, if the curve occurs suddenly, it is indicated that the loss coefficient is obtained using the following expression:

$$\xi_{SC} = 67.6x10^{-6}(\theta)^{2.17} \quad (42)$$

The above equation is also recommended to estimate the loss coefficient in sudden elbows.

On the other hand, according to Trueba (1986), the loss coefficient of a sudden curve is obtained through the following equation:

$$\xi_{SC} = 0.25\left(\frac{\theta}{90}\right)^{0.5} \quad (43)$$

where the angle $\theta$ must be from 0 to 90º.

According to Miller (1990), in the case of gradual curves that present smooth surfaces, relations $L_d/D \geq 30$ and values of $R_e$ = 10x10$^5$, the $\xi_{GC}$ must be obtained from a graph as a function of 0.5 ≤ $r/D$ ≤10 and 10 ≤ $\theta$ ≤ 180º; this graph is also suggested in CFE (1983) and Hager (2010). There various curves are exposed to obtain the $\xi_{GC}$, where it takes a minimum value of 0.02 and a maximum of 1.0. It is also shown that for any value of $\theta$, the coefficient decreases as the relation $r/D$ increases, originating the minimum values with $r/D$ between 1.0 and 3.0; from there, the value of $\xi_{GC}$ increases as $r/D$ also increases, but it fails to reach values like when $r/D$ < 1.0, where the maximum values are obtained for any value of $\theta$. However, if smooth walls or the values mentioned for $L_d/D$ and $R_e$, Miller (1990) and CFE (1983) establish that the loss coefficient obtained from the aforementioned graph must be corrected as follows:

$$\xi_{GC} = \xi_1 C_{R_e} C_{L_d} C_\varepsilon \quad (44)$$

where $\xi_1$ is the loss coefficient obtained from the graph for smooth pipes with relation $L_d/D$ ≥ 30 and values of $R_e$ = 10x10$^5$; $C_{R_e}$, $C_{L_d}$ and $C_\varepsilon$ are correction coefficients for $R_e$ ≠ 10x10$^5$, $L_d/D$ < 30 and roughness on the device walls, respectively. In the case of the coefficient $C_{R_e}$, when $r/D$ ≥ 1.0 or $\xi_1$ < 0.4, this coefficient is obtained from a graph in relation to $r/D$ ≥ 1 and 0.1 ≤ $R_e$ ≤ 100x10$^5$, where the value of $C_{R_e}$ decreases as the parameters that govern it increase. However, when $r/D$ < 1.0 the value $C_{R_e}$ must be determined with the following equation:

$$C_{R_e} = \frac{\xi_1}{\xi_1 - 0.2 C_{Re1} + 0.2} \quad (45)$$

where $C_{Re1}$ is the correction coefficient for $R_e$ ≠ 10x10$^5$ when $r/D$ ≥ 1.0 or $\xi_1$ < 0.4 (discussed previously). On the other hand, the coefficient $C_{L_d}$ is determined from a graph based on the coefficient $\xi_1$ and the relationship $L_d/D$, where its maximum values are presented in $L_d/D$ = 0, while its minimum values occur in $L_d/D$ from 1 to 4. Finally, if $r/D$ ≥ 1.0 and $R_e$ ≤ 10x10$^5$, the coefficient $C_\varepsilon$ is determined as follows:

$$C_\varepsilon = \frac{f_R}{f_L} \quad (46)$$

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where $f_R$ and $f_L$ are coefficients of resistance to friction in rough pipe and hydraulically smooth pipe, respectively. However, if $r/D \geq 1.0$ and $R_e >$ 10x10$^5$, the $C_\varepsilon$ is obtained with the previous equation, but obtaining the coefficients $f_R$ and $f_L$ with $R_e$ = 10x10$^5$.

Finally, if the curve is sudden, Miller (1990) and CFE (1983) indicate that the $\xi_{SC}$ is determined with the same procedure as for the gradual curve. For this, a graph is recommended to estimate the $\xi_{SC}$ as a function of 5 ≤ $\theta$ ≤ 120º, which applies to smooth pipes with $L_d/D \geq 30$ and $R_e$ = 10x10$^5$; otherwise, the coefficient must be corrected in the same way to the gradual condition. In this graph, the $\xi_{SC}$ increases from 0.02 to 1.5, as the value of $\theta$ increases from 5 to 120º, respectively.

Based on Franzini and Finnemore (1999), a graph is presented to estimate the coefficient of losses in gradual curves with $\theta$ = 90º, where $\xi_{GC}$ is governed by $r/D$ and $\varepsilon/D$; it should be noted that these results do not include friction loss in the curve. This graph shows that $\xi_{GC}$ increases as the ratio $\varepsilon/D$ increases for any value of $r/D$. However, the coefficient decreases and increases for each value of $\varepsilon/D$, when $r/D$ takes values from 1 to 7 and from 7 to 10, respectively, presenting the minimum values in $r/D$ = 7 and the maximum values in $r/D$ = 1. Therefore, the minimum value is 0.08 at $\varepsilon/D$ = 0 and the maximum is 0.90 at $\varepsilon/D$ = 0.01. Finally, it is openly recommended that for curves with $\theta$ = 22.5 and 45º, the $\xi_{GC}$ is approximately 40 and 80% of the estimated value for a curve with $\theta$ = 90º.

Mays (2001) recommends a table to estimate the loss coefficient in gradual curves with smooth surfaces and $\theta$ = 45 and 90º, as a function of $r/D$. In the 45º curves the $\xi_{GC}$ decreases from 0.10 to 0.09, when $r/D$ increases from 1 to 2, then increases from 0.09 to 1.20 as $r/D$ increases from 2 to 6. However, in the 90º curves the $\xi_{GC}$ decreases 0.35 to 0.16 and then increases from 0.16 to 0.21, when $\xi_{GC}$ increases from 1 to 4 and from 4 to 6, respectively. Finally, it is indicated that in sudden 90º curves, the $\xi_{SC}$ can take a value of 1.10, which is quite general.

For his part, Albers (2010) proposes a graph to estimate the $\xi_{GC}$ if $\theta$ = 90º, which is also applicable for elbows. There the $\xi_{GC}$ decreases as $r/D$ increases, reaching a maximum and minimum value of 1.0 and 0, when $r/D$ approaches 0 and 8, respectively. On the other hand, it is indicated that for a curve with $\theta$ = 45, 135 and 180º, the value obtained of the $\xi_{GC}$ in the graph for $\theta$ = 90º, will be proportional to 0.5, 1.5 and 2.0, respectively.

Hager (2010), in addition to proposing the Miller (1990) graph for $R_e$ = 10x10$^5$, also suggests another graph to estimate the $\xi_{GC}$ with $R_e \geq$ 10x10$^5$, which is also suggested in Ito (1960) and Blevins (1984). In this graph the $\xi_{GC}$ can be obtained for $\theta$ = 45, 90 and 180º, as a function of 0.5 ≤ $r/D$ ≤ 10. Although this graph is represented differently, the behavior of the $\xi_{GC}$ is similar to that mentioned in Miller (1990). For $\theta$ = 45, 90 and 180º, the minimum values of $\xi_{GC}$ are approximated to 0.09, 0.14 and 0.18, when $r/D$ = 1.5, 2.5 and 2.0, respectively; while the maximum values are close to 0.25, 0.6 and 0.6, when $r/D$ = 0.5, 0.65 and 0.75, respectively. On the other hand, if $R_e$<10x10$^5$, the $\xi_{GC}$ obtained in the mentioned graph, must be multiplied by (10x10$^5$/$R_e$)$^{1/6}$.

Based on Kast (2010), a graph made with experimental results is recommended to estimate the $\xi_{GC}$ with smooth or rough surfaces, 1 ≤ $r/D$ ≤ 10 and 15 ≤ $\theta$ ≤ 180º, which conduct flows with $R_e$ ≥ 1x10$^5$. This graph shows that the $\xi_{GC}$ for smooth surfaces increases as the $\theta$ also increases, while with respect to $r/D$ it presents different behaviors. For example, when the angle $\theta$ = 15 and 30º, the $\xi_{GC}$ remains constant near 0.03 and 0.06, respectively, for any value of $r/D$. However, when $\theta$ = 45º, the $\xi_{GC}$ decreases as $r/D$ increases, taking maximum and minimum values of 0.14 and 0.07 for $r/D$ = 1 and 10, respectively. Furthermore, if $\theta$ = 90º, the $\xi_{GC}$ presents a maximum value of approximately 0.22 at $r/D$ = 1, from where it decreases to a minimum value of close to 0.09 at $r/D$ = 6.5, then increases again to approximately 0.11 when $r/D$ = 10. Now if $\theta$ = 180º, the coefficient decreases from 0.32 to 0.13, then increases to 0.15 and decreases again until the minimum value of 0.11, finally increasing to 0.15; these values are presented when $r/D$ is 1, 2.8, 4, 6.5 and 10, respectively. Finally, it is shown that for rough curves with $\theta$ = 90º, the behavior of $\xi_{GC}$ is similar, but with greater magnitude than that of smooth curves with the same $\theta$. Said coefficient decreases from the maximum value of 0.50 to the minimum value of 0.18, from where it increases to 0.20; this occurs when $r/D$ = 1, 6.5 and 10, respectively. On the other hand, another graph is also exposed to obtain the $\xi_{GC}$ with smooth surfaces, $\theta$ = 90º, 2.26 ≤ $r/D$ ≤ 11.71 and 20 ≤ $R_e$ ≤ 2x10$^5$. In this it is shown that the $\xi_{GC}$ decreases as the $R_e$ increases for any relation $r/D$, presenting the maximum and minimum value close to 100 and 0.14, when $R_e$ = 20 and 2x10$^5$, respectively. However, for 250 ≤ $R_e$ ≤ 900, the value

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of $\xi_{GC}$ remains constant near 2.0. Finally, information is suggested in tables of the loss coefficient for standard curves or elbows in rough conditions with $\theta$ = 90º, both for bolted connections and those made of cast iron. For bolted connection devices with $r_i$ = 0 and $r_o$ ≥ 0, as in curves with $r_i$ ≥ 0 and $r_o$ = 0, a table is recommended for $D$ from 1.4 to 4.9 cm, where the $\xi_{SC}$ decreases from 1.2 to 0.51, from the $D$ minimum to maximum, respectively. Now, if the sudden curves are made of cast iron, Herning's (1966) data is recommended for $D$ from 5 to 50 cm, where the coefficient increases from 1.3 to 2.2, as the minimum $D$ increases to the maximum, respectively.

Menon and Menon (2010) present a table to estimate the loss coefficient of standard sudden curves, as a function of 0 < $\theta$ ≤ 90º and $D$ from 0.92 to 59.05 cm ($Dn$ from 0.5 to 24 inches ≈ from 1.27 to 60.96 cm), without taking into account the value of $R_e$. In this table it is indicated that the $\xi_{SC}$ increases as the value of $\theta$ increases, while it decreases when the value of $D$ increases. Therefore, the minimum value of $\xi_{SC}$ = 0.02, when $\theta$ ≈ 0º and $D$ = 60.96 cm, while the maximum is 1.62, when $\theta$ = 90º and $D$ = 1.27 cm.

In the document by Mataix (2010), tabulated values of the loss coefficient are recommended, which are applicable in gradual curves and elbows with $\theta$ = 90º. The value of $\xi_{GC}$ is smaller as the ratio $r/D$ increases; the maximum value of the coefficient is 0.80 when $r/D$ ≈ 0, while the minimum value is 0.16 with $r/D$ = 1.0.

Regarding Munson, Okiishi, Huebsch and Rothmayer (2013), it is established that the loss coefficient of a curve or elbow with $\theta$ = 90º, can be estimated by means of a graph as a function of $\varepsilon/D$ and $r/D$, with ranges from 0 to 0.01 and 1 to 10, respectively. In this graph, it is indicated that for any value of $\varepsilon/D$, the $\xi_{GC}$ decreases when $r/D$ goes from 1 to 7, where it reaches the minimum values, then increases until $r/D$ = 10 without exceeding the maximum values that occur at $r/D$ = 1. Therefore, in this case the minimum value of $\xi_{GC}$ ≈ 0.06, when $\varepsilon/D$ ≈ 0 and $r/D$ = 7, while the maximum value is 0.95 at $\varepsilon/D$ = 0.01 and $r/D$ = 1.

According to Sotelo (2013), it is established that the loss coefficient of a gradual curve with rough surfaces is determined by the following equation:

$$\xi_{GC} = C_a \frac{\theta}{90} \tag{47}$$

where $\theta$ can be from 0 to 90º; $C_a$ a is a coefficient is estimated with a graph of results Hofmann (1929) for flows with $R_e$ ≥ 2x10$^5$, where it is a function of the relations $r/D$ and $\varepsilon/D$, from 1 to 10 and 0 to 0.002, respectively. In this graph, the value of $C_a$ grows as the relation $\varepsilon/D$ increases for any value of $r/D$; however, in all the $\varepsilon/D$ relationships, this coefficient decreases and then increases, as $r/D$ increases from 1 to 7 and from 7 to 10, respectively. Therefore, the minimum value of $C_a$ occurs at $r/D$ = 7 and $\varepsilon/D$ ≈ 0, while the maximum at $r/D$ = 1 and $\varepsilon/D$ = 0.002.

In Gülich (2014) an equation is suggested to calculate the loss coefficient in a sudden curve, which is shown below:

$$\xi_{SC} = 1.2\left(\frac{\theta}{90}\right)^2 \tag{48}$$

where $\theta$ can take values from 0 to 90º. On the other hand, an equation is also suggested to estimate the loss coefficient of gradual curves, which is written as follows:

$$\xi_{GC} = C_{\varepsilon}\frac{0.23}{\left({}^{r}/_{D}\right)^{x}}\sqrt{\frac{\theta}{90}} \tag{49}$$

in which $C_{\varepsilon}$ is a roughness coefficient that is equal to unity on smooth surfaces, while on rough surfaces with $\varepsilon/D$ > 0.001 and $R_e$ > 0.4x10$^5$, said coefficient is equal to 2; $x$ is an exponent whose value is 2.5 and 0.5, when 0.5 ≤ $r/D$ ≤ 1.0 and $r/D$ > 1.0, respectively.

## Summary of methodologies for $\xi_C$

The Table 1 shows a summary of the methodologies discussed in the previous section, to estimate the coefficient of losses in simple curves installed in series with Z-shape, U-shape, L-shape and S-shape, where they are indicated important characteristics and the parameters considered in each one of them.

**Table 1**. Characteristics and parameters considered to estimate the loss coefficient in the analyzed methodologies.

| **Ref. (source)** | **Used fluid** | **Material** | ***D* (cm)** | ***r/D*** | **$\theta$° o shape** | **$R_e$ (x10⁵)** | **$L_u/L_d$ (x*D*)** | **Observations** |
|---|---|---|---|---|---|---|---|---|
| Brightmore (1907) | Water | Steel | 7.62<br>10.16 | 2.5 to 10 | 90 | I | NS | Graphics of $\xi_{GC}$ |
| Davis (1911) | Water | Steel | 5.08 | 5.0 to 20.5 | 90 | I | NS | Graphics of $\xi_{GC}$ |
| Balch (1913) | Water | Steel | 7.62 | 7.0 to 20 | 90 | I | NS | Graphics of $\xi_{GC}$ |
| Hofmann (1929) | Water | Steel | 4.32 | 1.0 to 10 | 90 | I | NS | $\xi_{GC}$ graphics for smooth and rough surfaces |
| Gibson (1930)<br>Weisbach (1855)<br>Brightmore (1907) | Water | Steel, wrought and cast iron | 3.17<br>7.62 and 10.16<br>15.40 | -<br>2.0 to 10<br>1.34 to 20 | 90 | I | NS | Equations (21-22) and tables for $\xi_{SC}$ and $\xi_{GC}$ |

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|---|---|---|---|---|---|---|---|---|
| Schoder (1908) | | | | | | | | |
| Vogel (1933) | Water | Steel | 15.24<br>20.32<br>25.40 | 1.0 to 3.0 | 90 | I | NS | Graphics of $\xi_{GC}$ |
| Beij (1938) | Water | Steel | 10.23 | 0.97 to 19.96 | 90 | 0.23 to 3.4 | 48.15/ 168.12 | Tables and graphs of $\xi_{GC}$, $Dn$ = 4 in, results compared with: Brightmore (1907) Davis (1911) Balch (1913) Hofmann (1929) Vogel (1933) |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Daugherty e Ingersoll (1954)<br>Pigott (1950) | Water | NS | NS | AV | 0 to 180 | I*f* | NS | Equation (23) for gradual curves that depends on *f*. |
| Ito (1956) | Water | Cast brass | 3.5 | 28.4 and 108<br>7.6 to 29.2<br>5.8 and 9.05<br>5.2 and 7.4 | 45<br>90<br>135<br>U | 0.1 to 3.0 | NS | $\xi_{GC}$ graphs for long radius curves and smooth surfaces |
| Ito (1960) | Water | Cast brass | 3.5 | 1.84 to 7.3<br>1.0 to 3.3<br>1.84 and 3.26 | 45<br>90<br>U | 0.2 to 4.0 | 153.8/ >71.6 | Graphs and equations (7-12) of $\xi_{GC}$ of Re for smooth surfaces. Results compared with: Weisbach (1855)<br>Hofmann (1929)<br>Richter (1930) |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | Wasielewski (1932)<br>Pigott (1950)<br>Pigott (1957) |
| Idel'chik (1966)<br>Abramovich, (1935)<br>Nekrasov (1954)<br>Idel'chik (1953)<br>Evdomikov (1940) | NS | NS | NS | ≥0.5<br>0.5 to 1.5*<br>1.5 to 50* | 0 to 180(U) | ≥ 2.0<br>≥ 0.03*<br>≥ 0.03* | NS | Tables, graphs and equations (24-30) of $\xi_C$, for smooth and rough surfaces* |
| Idel'chik (1966)<br>Aronov (1950)<br>Adler (1934)<br>White (1929) | NS | NS | NS | ≫1.5 | 0 to 90 | 0.0005 to 0.2 | NS | Tables, graphs and equations (31-34) of $\xi_C$ for smooth surfaces |
| Idel'chik (1966) | NS | NS | NS | 0 | 0 to 180 | ≥0.4<br>≥0.003* | NS | Tables, graphs and equations (35-37) of $\xi_{SC}$ for smooth and |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Abramovich, (1935)<br>Richter (1930)<br>Richter (1936)<br>Weisbach (1855)<br>Schubart, (1926) | | | | | | | | rough surfaces* |
| Idel'chik (1966) | NS | NS | NS | ≥0.5 | L<br>U<br>Z | ≥2.0<br>≥0.03* | NS | Tables, graphs and equations (38-39) of $\xi_{GC}$ for smooth and rough surfaces * |
| Chen-Tzu (1969) | Water | PVC | 5.25 | 1.59 | 90H*<br>90V** | 0.28 to 1.86 | 30/36 | Graphs of $\xi_{GC}$, $T$ = 21.1 °C, study of horizontal (H) and vertical (V) curves |
| King *et al*. (1980); Brater *et al*. (1996) | NS | NS | NS | 1 to 20 | 45<br>90<br>180 | ≥2.0 | NS | Tables of $\xi_{GC}$ |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Beij (1938) | | | | | | | | |
| USACE (1980); Sotelo (2013) Anderson, (1947) Hofmann (1929) Wasielewski (1932) Kirchbach (1929) Schubart (1929) | Water | NS | NS | 1 to 10 0* | 0 to 90 | ≥2.0 0.2 to 2.25* | NS | Graphs of $\xi_{GC}$ and $\xi_{SC}$* |
| Crane Co. (1982) Pigott (1950) Beij (1938) Kirchbach, (1929) | Water | Steel | 1.27 to 60.96 | 1.0 to 20 | 90 180 0 to 90* | I | NS | Tables and graphs of $\xi_{SC}$* and $\xi_{GC}$. Steel and flanged end devices |
| Turian *et al*. (1983) Hsu (1981) | Water-Solids | Steel | 2.54 5.08 | 1.0 to 8.33 | 90-S 45 90-G | 0.1 to 0.8 | NS | Tables and graphs of $\xi_{SC}$ and $\xi_{GC}$. Flow of water with |

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|---|---|---|---|---|---|---|---|---|
| | | | | 1.0 to 11.1 | U | 0.2 to 1.7 | | glass beads. Results compared with Crane Co. (1982) |
| Gontsov *et al*. (1984) | Air | Organic Glass | 20.6 | 1.0 | 90 | 0.9 to 4.0 | 35/25 | Tables and equations (13-15) of $\xi_{GC}$ for smooth surfaces. Results compared with Ito (1960) |
| SARH (1984) | Water | NS | NS | 1.0 to 10 | 0 to 180 | I | NS | Tables of $\xi_{GC}$. |
| Pashkov y Dolqachev (1985) | NS | NS | ≤3.0 | I | 0 to 180 | I | NS | Tables of $\xi_{GC}$ and equation (40) of $\xi_{SC}$* |
| USBR (1985) Beij (1938) | Water | Steel | NS | 1 to 10 | 0 to 120 | I | NS | Graphics of $\xi_{GC}$ |
| Simon (1986) | NS | NS | NS | AV 0* | 0 to 180 | I | NS | Equations (41-42) of $\xi_{SC}$* and $\xi_{GC}$. |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Trueba (1986) | NS | NS | NS | 0 | 0 to 90 | I | NS | Equation (43) of $\xi_{SC}$ |
| Miller (1990); CFE (1983) | NS | NS | NS | 0.5 to 10<br>0* | 10 to 180<br>5 to 20* | 0.1 to 100 | NS / ≥0 | Graphs and equations (44-46) of $\xi_{GC}$ and $\xi_{SC}$* for smooth and rough surfaces. Corrections for Re, $L_d$ and $\varepsilon$ are considered. |
| Franzini & Finnemore (1999) | NS | NS | NS | 1.0 to 10 | 22.5, 45 and 90 | I | NS | Graph of $\xi_{GC}$, where it is also governed by $\varepsilon/D$ |
| Mays (2001) | NS | NS | NS | 1.0 to 6.0 | 45 and 90<br>90* | I | NS | Table of $\xi_{GC}$ and $\xi_{SC}$* for smooth surfaces |
| Albers (2010) | NS | NS | NS | 0 to 8.0 | 45, 90, 135 and 180 | I | NS | Graph of $\xi_{GC}$. It is also applicable for elbows. |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Hager (2010) Miller (1990) Ito (1960) Blevins (1984) | NS | NS | NS | 0.5 to 10 | 45 to 180 | ≥10, <10 | NS | Graphics of $\xi_{GC}$ |
| Kast (2010) | NS | Iron | 1.4 to 50* | 1.0 to 10 0* | 15 to 180 0 to 90* | ≥0.04 | NS | Tables and graphs of $\xi_{SC}$* and $\xi_{GC}$ for smooth and rough surfaces. Tables of $\xi_{SC}$* are also exposed for $r_i$ ≥ 0 and $r_o$ ≥ 0 |
| Menon y Menon (2010) | NS | NS | 0.92 to 59.05 | 0 | 0 to 90 | I | NS | $\xi_{SC}$ tables for standard curves |
| Mataix (2010) | NS | NS | NS | 0 to 1.0 | 90 | I | NS | $\xi_{SC}$ tables that are also applicable in elbows |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Munson *et al*. (2013) | NS | NS | NS | 1.0 to 10 | 90 | I | NS | $\xi_{SC}$ tables that are also applicable in elbows, where it is also governed by $\varepsilon/D$ |
| Sotelo (2013) Hofmann, (1929) | NS | NS | NS | 1.0 to 10 | 0 to 90 | ≥2.0 | NS | Graph and equation (47) of $\xi_{SC}$ for rough surfaces, where it is also governed by $\varepsilon/D$ |
| Gülich (2014) | NS | NS | NS | 0* ≥0.5 | 0 to 90 | I | NS | Equations (48*-49) of $\xi_{SC}$* and $\xi_{GC}$ for smooth and rough surfaces |
| Villegas-León *et al*. (2016) | NS | NS | NS | 0* 1.0 to 10 | 0 to 90* 5 to 90 | I | NS | Equations (16-20) of $\xi_{SC}$* and $\xi_{GC}$ |

NS = not specified, I=independent of its value, AV =any value is valid, I*f* = implicit in the value of the coefficient *f*

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The above table allows contrasting the fluids and types of materials of the curves used in the investigations of the loss coefficient in curves, the considered values of $D$, $r/D$, $\theta$, $R_e$, $L_u$ and $L_d$, as well as observations of the results presented. These data indicate that the fluid most used in the tests is water and that the type of material most analyzed is steel and iron. It can also be observed that investigations have been carried out up to $D$ ≈ 60 cm (Crane Co., 1982; Menon & Menon, 2010), but most have been in small $D$ close to 10 cm (4 inches). The largest investigated $r/D$ relationship is 108 cm (Ito, 1956), however, the most analyzed is in the range $r/D \leq 10$. Regarding the angle $\theta$ and type of shape, it is noted that the most studied curves they are of gradual condition with $\theta$ = 90º and U-shaped structures with $L_s$ = 0; from the values obtained from the loss coefficient with $\theta$ = 90º, some authors such as Franzini and Finnemore (1999), and Albers (2010), recommend inferring values for different angles, which can lead to large errors in the estimation of losses of energy. On the other hand, it is illustrated which methodologies consider the value of $R_e$, its validity range and which ones consider it independent to obtain the coefficient of the curves; a factor that is important to estimate a coefficient that is closer to reality, since in documents such a Turian *et al*. (1983), Kast (2010) and Hager (2010), it can be noticed that it affects its value in a decreasing way as $R_e$ increases. Another important parameter that is confronted is up to which value of $L_d$ the energy losses were analyzed, since it has been shown that up to approximately $L_d$ = 50$D$ (Ito, 1956; Ito, 1960), it still affects the energy loss caused by the curves of pipes. In the last column important observations are presented, where it is indicated which coefficient is determined ($\xi_{SC}$, $\xi_{GC}$), the form of presentation of results (graphs, tables or equations), as well as for which type of surfaces they are valid (smooth or rough). Finally, it should be noted that several methodologies do not refer to the type of material and diameter used in the studies, which implies using them in a general way, and may differ substantially in actual losses.

## Conclusions

- A literature review of experimental research and reported methodologies was carried out, which allow estimating the coefficient of losses in curves of pipes under turbulent flow with sudden and gradual conditions, which can be presented in a simple way and installed in series, to form structures with shapes type L, S, U and Z. From this, organized and homogenized information was presented, which facilitates comparing the scope of the methodologies, as well as the behavior of the loss coefficient before the parameters govern it.
- Through research and the methodologies analyzed in the literature, different values of the loss coefficient are obtained in the devices studied, which may be due to differences adopted in the experiments, such as: the

device's manufacturing material, the roughness height of the interior walls, diameter, radius of curvature, deflection angles, temperature and flow velocities, type of fluid used, types of connection, accuracy of measuring devices, and lengths of upstream and downstream pipes down the devices. However, all these characteristics allow a choice more in line with a particular case, of the methodology or the values to be used in estimating the loss coefficient.

- The loss coefficients reported in pipe curves may or may not include friction loss over the length of the curve ($h_f$), affecting the value of the coefficient and even more so when the devices have a long radius of curvature. In addition to this, the distance considered downstream of the device for the evaluation of the loss due to flow disturbance ($h_d$), also influences the loss coefficient, which has been shown that $L_d$ can reach up to $50D$.
- In the investigations it was shown that the loss coefficient has dependence on the angle $\theta$, the relation $r/D$, the value of $R_e$ and the relation $\varepsilon/D$. However, often in literature methodologies, the coefficient is only reported as a function of $\theta$ and $r/D$, but the most excluded parameter is surface roughness or the $\varepsilon/D$ ratio. In general, the coefficient of losses increased according to the increase in $\theta$ and $\varepsilon/D$, but a decrease in relation to the growth of $R_e$. In the gradual condition devices, the coefficient increased as the ratio $r/D$ decreased, while, in the four forms with curves in series, the coefficient increased according to the increase in the separation length between the devices ($L_s$).

- The most studied devices and of which more information on the loss coefficient is presented, are the curves with $\theta$ = 45, 90 and 180º. However, the most researched of them due to its frequent use is the device with $\theta$ = 90º, in which some authors infer the loss coefficient for curves of 45 and 180º of deflection. Regarding the material of the device, the direction changes with the most research are those of steel, while the least analyzed are the modern ones (plastic, molded and forged), which tend to provide lower values of the loss coefficient due to their low roughness.
- Finally, it is concluded that the information from the works of Ito (1959, 1960) is the most suggested in the literature to estimate the loss coefficient in gradual devices, while in sudden devices the data is recommended more frequently from the work of Kirchbach (1929). However, where it provides extensive information on the loss coefficient of all the devices studied with respect to their dependent parameters, is in the resistance manual by Idel'chik (1966).

## References

Abramovich, G. N. (1935). Aerodinamika mestnykh soprotivlenii (Aerodynamics of local resistances). *Sbornik Po Promyshlennoi Aerodinamike, Trudy*, 211, 65-150.

Acero, M. F., & Rodríguez, D. (May, 2008). Determinacion experimental del coeficiente de pérdidas menores km en accesorios para agua potable de PVC. *XVIII Seminario Nacional de Hidráulica e Hidrología,* *Sociedad Colombiana de Ingenieros*. Recovered from https://es.scribd.com/document/48818587/02-Determinacion-Experimental-de-km-Accesorios-PVC-P

Adler, M. (1934). Strömung in gekrümmten Röhren. *Zeitschrift für Angewandte Mathematik und Mechanik*, 14, 257-275.

Albers, P. (2010). Motion control in offshore and dredging. Recovered from https://doi.org/10.1007/978-90-481-8803-1

Anaya-Durand, A. I., Cauich-Segovia, G. I., Funabazama-Bárcenas, O., & Gracia-Medrano-Bravo, V. A. (2014). Evaluación de ecuaciones de factor de fricción explícito para tuberías. *Educación Química*, 25(2), 128-134. Recovered from https://doi.org/10.1016/S0187-893X(14)70535-X

Anderson, A. G. (1947). Fluid flow diversion: A summary and bibliography of literature. In: *St. Anthony Falls Hydraulic Laboratory* (1). Minneapolis, USA: University of Minnesota.

Aronov, I. Z. (1950). *Teploobmen i gidravlicheskoe soprotivlenie v izognutykh trubakh (Heat Exchange and Hydraulic Resistance in Bent Pipes)* (Doctorate's Thesis). Kiev, Ukraine: Kiev Polytechnic Institute.

Arun, G., Kumaresh, S. P., Natarajan, S., & Kulasekharan, N. (2020). Study of flow behaviour in sharp and mitred pipe bends. *Materials Today: Proceedings*, 27(3), 2101-2108. Recovered from https://doi.org/10.1016/j.matpr.2019.09.076

Balch, L. R. (1913). Investigation of hydraulic curve resistance, experiments with three-inch pipe. *Bulletin of the University of*

*Wisconsin*, 403(7), 253.

Bariviera, D., Frizzone, J. A., & Rettore, O. (2014). Dimensional analysis approach to estimate local head losses in microirrigation connectors. *Irrigation Science*, 32, 169-179. Recovered from https://doi.org/10.1007/s00271-013-0424-y

Beij, K. H. (1938). Pressure losses for fluid flow in 90° pipe bends. *Journal of Research of the National Bureau of Standards*, 21(1), 1-18. Recovered from https://doi.org/http://dx.doi.org/10.6028/jres.021.001

Blevins, R. D. (1984). *Applied fluid dynamics handbook*. New York, USA: Van Nostrand Reinhold.

Brater, E. F., King, H. W., Lindell, J. E., & Wei, C. Y. (1996). *Handbook of Hydraulics* (7th ed.). New York, USA: McGraw-Hill Professional.

Brightmore, A. W. (1907). Loss of pressure in water flowing through straight and curved pipes. *Minutes of the Proceedings of the Institution of Civil Engineers*, 169(1907), 315-336. Recovered from https://doi.org/10.1680/imotp.1907.17203

CFE, Comisión Federal de Electricidad. (1983). *Manual de diseño de obras civiles, Hidrotecnia: Hidráulica, A.2.3 Conducciones a presión*. México, DF, México: Comisión Federal de Electricidad, Instituto de Investigaciones Eléctricas.

Chen-Tzu, Y. (1969). *Loss of pressure in water flowing through 90° pipe bend* (Kansas State University). Recovered from https://archive.org/stream/lossofpressurein00yuch/lossofpressurein

00yuch_djvu.txt

Chowdhury, R. R., Biswas, S., Alam, M. M., & Islam, A. K. M. S. (2016). Turbulent flow analysis on bend and downstream of the bend for different curvature ratio. *AIP Conference Proceedings*, 1754, 1-6. Recovered from https://doi.org/10.1063/1.4958380

Crane Co. (1982). Flow of fluids throuh valves, fittings and pipe. In: *Technical Paper No. 410*. New York, USA: Crane Company.

Daneshfaraz, R., Rezazadehjoudi, A., & Abraham, J. (2018). Numerical investigation on the effect of sudden contraction on flow behavior in a 90-degree bend. *KSCE Journal of Civil Engineering*, 22(2), 603-612. Recovered from https://doi.org/10.1007/s12205-017-1313-3

Dang, Z., Yang, Z., Yang, X., & Ishii, M. (2018). Experimental study of vertical and horizontal two-phase pipe flow through double 90 degree elbows. *International Journal of Heat and Mass Transfer*, 120, 861-869. Recovered from https://doi.org/10.1016/j.ijheatmasstransfer.2017.11.089

Daugherty, R. L., & Ingersoll, A. C. (1954). *Fluid Mechanics, with Engineering Applications* (6th ed.). Daugherty, R. L. & Ingersoll, A. C. (eds.). New York, USA: McGraw-Hil.

Davis, G. J. (1911). Investigation of hydraulic curve resistance. *Bulletin of the University of Wisconsin*, 403(6), 115.

Du, X., Wei, A., Fang, Y., Yang, Z., Wei, D., Lin, C., & Jin, Z. (2020). The effect of bend angle on pressure drop and flow behavior in a corrugated duct. *Acta Mechanica*. Recovered from https://doi.org/10.1007/s00707-020-02716-5

Dutta, P., & Nandi, N. (2015). Study on pressure drop characteristics of single phase turbulent flow in pipe bend for high Reynolds Number. *ARPN Journal of Engineering and Applied Sciences*, 10(5), 2221-2226. Recovered from http://www.arpnjournals.com/jeas/research_papers/rp_2015/jeas_0315_1746.pdf

Elbatran, A. H., Yaakob, O. B., Ahmed, Y. M., & Shabara, H. M. (2015). Operation, performance and economic analysis of low head micro-hydropower turbines for rural and remote areas: A review. *Renewable and Sustainable Energy Reviews*, 43, 40-50. Recovered from https://doi.org/10.1016/j.rser.2014.11.045

Evdomikov, I. F. (1940). Opyty po otsasyvaniyu pogranichnogo sloya v aerodinamicheskikh trubakh bol’shikh skorostei (Experiments on the suction of the boundary layer in high-velocity wind tunnels). *Trudy TsAGI*, 506, n.p.

Franzini, J. B., & Finnemore, E. J. (1999). *Mecánica de fluidos con aplicaciones en ingeniería* (9th ed.). Madrid, España: McGraw-Hill / Interamericana de España.

Friman, M., & Levy, A. (2019). Bends pressure drop in hydraulic conveying. *Advanced Powder Technology*, 30(8), 1484-1493. Recovered from https://doi.org/10.1016/j.apt.2019.04.025

Fuentes, O. A., & Rosales, I. L. (2004). Estimación de pérdidas locales de energía en tomas domiciliarias de agua potable. *Ingeniería Hidráulica* *en México*, 19(1), 65-73. Recovered from file:///C:/Users/Jose Juan/Downloads/1009-1109-1-PB.pdf

Gibson, A. H. (1930). *Hydraulics and its applications* (4th ed.). New York, USA: D. Van Nostrand Company.

Gontsov, N. G., Marinova, O. A., & Tananaev, A. V. (1984). Turbulent flow around a bend in a circular pipe. *Hydrotechnical Construction*, 18(12), 596-602. Recovered from https://doi.org/10.1007/BF01440641

Gülich, J. F. (2014). Fluid dynamic principles. In: *Centrifugal pumps* (3rd ed.) (pp. 1-41). Recovered from https://doi.org/10.1007/978-3-642-40114-5_1

Haase, D. (1953). *Allgemeine Wärmetechnik*. *Nos*(11–12), n.p.

Hager, W. H. (2010). Waste water hydraulics, theory and practice. In: *Springer Science and Business Media* (2nd. ed.). Recovered from https://doi.org/10.1007/978-3-642-11383-3

Herning, F. (1966). *Stoffströme in Rohrleitungen* (4t ed.). Düsseldorf, Germany: VDI.

Herwig, H., & Wenterodt, T. (2011). Second law analysis of momentum and heat transfer in unit operations. *International Journal of Heat and Mass Transfer*, 54, 1323-1330. Recovered from https://doi.org/10.1016/j.ijheatmasstransfer.2010.11.055

Hofmann, A. (1929). Der Verlust in 90° Rohrkrümmern mit gleiclibleibendem Kreisquerschnitt. *Mitteilungen Des Hydraulischen Instituts Der Technischen Hochschule München*, 3, 45-67.

Hsu, B. E. (1981). *Flow of fine particle suspensions in bends, fittings and valves* (Texas Tech University). Recovered from https://ttu-ir.tdl.org/ttu-ir/bitstream/handle/2346/19704/31295002382256.pdf?sequence=1&isAllowed=y

Idel'chik, I. E. (1966). *Handbook of hydraulic resistance, coefficients of local resistance and of friction* (No. 6630). Recovered from https://doi.org/AEC-tr- 6630

Idel'chik, I. E. (1953). K voprosu o vliyanii chisla Re i sherokhovatosti na soprotivienie izognutykh kanalov (The Influence of the Reynolds Number and the roughness on the resistance of bent channels). *Promyshlennaya Aerodinamika*. 4, 177-194.

Ito, H. (1960). Pressure losses in smooth pipe bends. *Journal of Basic Engineering*, 82(1), 131-140. Recovered from https://doi.org/10.1115/1.3662501

Ito, H. (1959). Friction factors for turbulent flow in curved pipes. *Transactions of the American Society of Mechanical Engineers, Series D, Journal of Basic Engineering*, 81, 123-134.

Ito, H. (1956). On the pressure losses for turbulent flow in smooth pipe bends. In: *Reports of The Institute of High Speed Mechanics*, 6, 55-102.

Jia, W., & Yan, J. (2020). Pressure drop characteristics and minimum pressure drop velocity for pneumatic conveying of polyacrylamide in a horizontal pipe with bends at both ends. *Powder Technology*, 372, 192-203. Recovered from https://doi.org/10.1016/j.powtec.2020.06.004

Kast, W. (2010). L1 Pressure drop in single phase flow. In: e. V. VDI (Ed.), *VDI Heat Atlas* (2nd ed.) (pp. 1053-1116). Recovered from https://doi.org/10.1007/978-3-540-77877-6_70

Kilkovsky, B., Jegla, Z., & Stehlik, P. (2011). Comparison of different methods for pressure drop calculation in 90° and 180° Elbows. *Chemical Engineering Transactions*, 25(1), 243-248. Recovered from https://doi.org/10.3303/CET1125041

King, H. W., Wisler, C. O., & Woodburn, J. G. (1980). *Hidráulica*. México, DF, México: Trillas.

Kirchbach, H. (1929). Der Energieverlust in Kniestücken (The energy loss in elbows, in German). *Mitteilungen Des Hydraulischen Instituts Der Technischen Hochschule*, 3, 43-64.

Liu, S. H., Xue, J., & Fan, M. (2013). The calculation of mechanical energy loss for incompressible steady pipe flow of homogeneous fluid. *Journal of Hydrodynamics*, 25(6), 912-918. Recovered from https://doi.org/10.1016/S1001-6058(13)60440-0

Lourenco, A., & Xin, G. (2017). *Design and development of an experimental apparatus for the study of minor loss of the mini-scale fittings* (University of Macau). Recovered from http://oaps.umac.mo/handle/10692.1/152

Maria-Di, F. (2000). Design and off design pipe network geothermal power plant analysis with power pipe simulator. *Energy Conversion* *and Management*, 41, 1223-1235. Recovered from https://doi.org/10.1016/S0196-8904(99)00177-6

Mataix, C. (2010). *Mecánica de fluidos y máquinas hidráulicas* (2nd ed.). México, DF, México: Alfaomega Editorial.

Mays, L. W. (2001). *Water Resources Engineering*. New York, USA: John Wiley & Sons, Inc.

Menon, E. S., & Menon, P. S. (2010). Pressure loss through piping systems. In: Menon, E. S. & Menon, P. S. (eds.). *Working Guide to Pumps and Pumping Stations* (pp. 69-111). Recovered from https://doi.org/https://doi.org/10.1016/B978-1-85617-828-0.00004-4

Miller, D. S. (1990). *Internal Flow Systems* (2nd ed.). Cranfield-Bedford, UK: BHRA.

Munson, B. R., Okiishi, T. H., Huebsch, W. W., & Rothmayer, A. P. (2013). Fundamentals of fluid mechanics. In: Fowley, D., Welter, J., & Holm, K. (eds.). *Biofluid Mechanics* (7th ed.). Recovered from https://doi.org/10.1016/B978-0-12-381383-1.00002-3

Nasir, B. A. (2014). Design considerations of micro-hydro-electric power plant. *Energy Procedia*, 50, 19-29. Recovered from https://doi.org/10.1016/j.egypro.2014.06.003

Nekrasov, B. B. (1954). *Gidravlika (Hydraulics)*. Russia: VVA.

Pashkov, N. N., & Dolqachev, F. M. (1985). *Hidráulica y máquinas hidráulicas*. Moscú, Rusia: Mir Moscú.

Perumal, K., & Ganesan, R. (2016). CFD modeling for the estimation of pressure loss coefficients of pipe fittings: An undergraduate project. *Computer Applications in Engineering Education*, 24(2), 180-185. Recovered from https://doi.org/10.1002/cae.21695

Pigott, R. J. S. (1957). Losses in Pipe and Fittings. *Transactions of the American Society of Mechanical Engineers*, 79, 1767-1783.

Pigott, R. J. S. (1950). Pressure losses in tubing, pipe, and fittings. *Transactions of the American Society of Mechanical Engineers*, 72, 679-688.

Reghunathan, R., Son, S. W., Suryan, A., & Kim, H. D. (2019). Study on reduction in pressure losses in pipe bends using guide vanes. *Journal of Visualization*, 22(4), 795-807. Recovered from https://doi.org/10.1007/s12650-019-00561-w

Richter, G. (1936). *Gidravlika truboprovodov (Hydraulics of Pipelines)*. Moscow, Russia: ONTI.

Richter, H. (1930). Der Druckabfall in gekrümmten glatten Rohrleitungen. *Forschungsarbeiten Auf Dem Gebiete Des Ingenieurwesens, VDI*, 338, 30-47.

Saldarriaga, J. G. (2016). *Hidráulica de tuberías: abastecimiento de agua, redes y riegos* (3ª. ed.). Recovered from http://books.google.com/books?id=-76BPgAACAAJ&pgis=1

SARH, Secretaría de Agricultura y Recursos Hidráulicos. (1984). *Obras de toma para presas de almacenamiento*. México, DF, México: Dirección General de Obras Hidráulicas y de Ingeniería Agrícola para el

Desarrollo Rural, Secretaría de Agricultura y Recursos Hidráulicos.

Schmandt, B., Iyer, V., & Herwig, H. (2014). Determination of head change coefficients for dividing and combining junctions: A method based on the second law of thermodynamics. *Chemical Engineering Science*, 111, 191-202. Recovered from https://doi.org/10.1016/j.ces.2014.02.035

Schoder, E. W. (1908). Loss coefficient experiments in pipe bends. *Transactions of the American Society of Civil Engineers*, 34, 416-445.

Schubart, W. (1929). Der Verlust in Kniestücken bei glatter und rauher Wandung (The loss in elbows with smooth and rough wall, in German). *Mitteilungen Des Hydraulischen Inst. Der Technischen Hochschule*, 3, 81-99.

Sesma, J., Molina-Martínez, J. M., Cavas-Martínez, F., & Fernández-Pacheco, D. G. (2015). A mobile application to calculate optimum drip irrigation laterals. *Agricultural Water Management*, 151, 13-18. Recovered from https://doi.org/10.1016/j.agwat.2014.09.026

Shames, I. H. (1995). *Mecánica de fluidos* (3a ed.). Bogotá, Colombia: McGraw-Hill Book Interamericana.

Simon, A. L. (1986). *Hidráulica práctica*. México, DF, México: Limusa.

Sotelo, G. A. (2013). *Hidráulica general*. Recovered from https://doi.org/10.1017/CBO9781107415324.004

Tripathi, N. M., Portnikov, D., Levy, A., & Kalman, H. (2019). Bend pressure drop in horizontal and vertical dilute phase pneumatic conveying systems. *Chemical Engineering Science*, 209, 115228.

Recovered from https://doi.org/10.1016/j.ces.2019.115228

Trueba, C. S. (1986). *Hidráulica* (23a ed.). México, DF, México: Continental S. A.

Turian, R. M., Hsu, F. L., & Selim, M. S. (1983). Friction losses for flow of slurries in pipeline bends, fittings, and valves. *Particulate Science and Technology*, 1(4), 365-392. Recovered from https://doi.org/10.1080/02726358308906383

USACE, United States Army Corps of Engineer. (1980). *Engineering and design: Hydraulic design of reservoir outlet works*. Recovered from http://www.publications.usace.army.mil/Portals/76/Publications/EngineerManuals/EM_1110-2-1602.pdf

USBR, United States Bureau of Reclamation. (1985). *Diseño de presas pequeñas* (13a ed.). México, DF, México: Continental S. A.

Villegas-León, J. J., López-Lambraño, A. A., Morales-Nava, J. G., Pliego-Díaz, M., Fuentes, C., & López-Ramos, A. (2016). Equations to determine energy losses in sudden and gradual change of direction. *Recent Advances in Fluid Dynamics with Environmental Applications*, 4, 465-475. Recovered from https://doi.org/10.1007/978-3-319-27965-7

Vogel, H. D. (1933). *Head losses in various types of pipe bends*. Reston, USA: The U.S. Army Corps of Engineers Waterways Experiment Station.

Wasielewski, R. (1932). Verluste in glatten Rohrkrümmern mit kreisrundem Querschnitt bei weniger als 90° Ablenkung. *Mitteilungen*

*Des Hydraulischen Instituts Der Technischen Hochschule München*, 5, 53-57.

Weisbach, J. (1855). *Experimentelle Hydraulik*. Freiberg, Germany: n.d.

White, C. M. (1929). Streamline flow through curved pipes. *Proceedings of the Royal Society of London.Series A, Containing Papers of a Mathematical and Physical Character*, 123(792), 645-663. Recovered from https://doi.org/10.1098/rspa.1929.0089

White, F. M. (2008). *Mecánica de fluidos* (6a ed.). Madrid, España: McGraw-Hill.

Yildirim, G., & Singh, V. P. (2010). A MathCAD procedure for commercial pipeline hydraulic design considering local energy losses. *Advances in Engineering Software*, 41, 489-496. Recovered from https://doi.org/10.1016/j.advengsoft.2009.10.007

Yoo, D. H., & Singh, V. P. (2010). Explicit design of commercial pipes with secondary losses. *Journal of Hydro-Environment Research*, 4(1), 37-45. Recovered from https://doi.org/10.1016/j.jher.2009.12.003

Yuce, M., & Muratoglu, A. (2015). Hydrokinetic energy conversion systems: A technology status review. *Renewable and Sustainable Energy Reviews*, 43, 72-82. Recovered from https://doi.org/10.1016/j.rser.2010.06.016

Zmrhal, V., & Schwarzer, J. (2009). Numerical simulation of local loss coefficients of ventilation duct fittings. *Eleventh International IBPSA Conference*, July 27-30, 2009, 1761-1766. Recovered from http://www.ibpsa.org/proceedings/BS2009/BS09_1761_1766.pdf