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Articles

Empirical analysis of hydraulic jump roller length in sloped composite channels with different roughness coefficients

Análisis empírico de la longitud de rodillos de salto hidráulico en canales compuestos inclinados con diferentes coeficientes de rugosidad

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Abstract

The hydraulic jump is a fundamental transitional phenomenon between supercritical and subcritical flow in open channels, playing a pivotal role in stabilizing hydraulic systems, dissipating energy, and preventing scour downstream of hydraulic structures. This study investigates the combined effects of channel slope and major bed roughness in a rectangular compound channel on hydraulic jump characteristics. Laboratory experiments were conducted using a 10-meter flume equipped with adjustable longitudinal slopes and roughened surfaces of varying diameters. The study focused on determining the relative roller length (Lr/h_1) under different upstream Froude numbers, slopes, and roughness conditions. Results show that the relative roller length increases with the Froude number but decreases with increasing bed roughness. A global predictive equation was developed, integrating Froude number, slope, and roughness, which demonstrated excellent agreement with experimental data ($R^2 = 0.9985$) and low error margins. This confirms the robustness of the model and its potential for practical application in the design of stilling basins and hydraulic energy dissipation systems.

Keywords: Hydrodynamics, Fluid mechanics, Hydraulic engineering, Hydraulic structures, Canals, Equations, Statistical analysis.

Resumen

El salto hidráulico es un fenómeno de transición fundamental entre el flujo supercrítico y subcrítico en canales abiertos, que desempeña un papel fundamental en la estabilización de los sistemas hidráulicos, la disipación de energía y la prevención de la socavación aguas abajo de las estructuras hidráulicas. Este estudio investiga los efectos combinados de la pendiente del canal y la rugosidad principal del lecho en un canal compuesto rectangular sobre las características del salto hidráulico. Se realizaron experimentos de laboratorio utilizando un canal de 10 metros equipado con pendientes longitudinales ajustables y superficies rugosas de diferentes diámetros. El estudio se centró en determinar la longitud relativa del rodillo (Lr/h_1) bajo diferentes números de Froude, pendientes y condiciones de rugosidad aguas arriba. Los resultados muestran que la longitud relativa del rodillo aumenta con el número de Froude, pero disminuye con el aumento de la rugosidad del lecho. Se desarrolló una ecuación predictiva global que integra el número de Froude, la pendiente y la rugosidad, la cual demostró una excelente concordancia con los datos experimentales ($R^2 = 0,9985$) y bajos márgenes de error. Esto confirma la robustez del modelo y su potencial para la aplicación práctica en el diseño de cuencas amortiguadoras y sistemas de disipación de energía hidráulica.

Palabras clave: Hidrodinámica, Mecánica de fluidos, Ingeniería hidráulica, Estructura hidráulica, Canal, Ecuación, Análisis estadístico.

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1. Introduction

The transition of flow from a supercritical to a subcritical state in open channels is characterized by the hydraulic jump, a highly turbulent phenomenon essential in hydraulic engineering for stabilizing flow regimes, recovering head, and dissipating surplus kinetic energy (Rajaratnam, 1967); (Chanson, 2015). Early theoretical and experimental investigations, primarily focusing on horizontal, rectangular channels, established the classical momentum equations for jump analysis (Rajaratnam, 1965) ;(Hager & Bretz, 1987). More recent studies have continued to refine the analysis of jumps in simple geometries (Alghwail, 2021); (Chen-Feng Li, 1995); (Simsek et al., 2023); (Nandi et al., 2020); (Hafnaoui & Debabeche, 2023).

However, in many real-world applications, channels are constructed with a non-zero slope, which significantly alters the flow dynamics and the characteristics of the hydraulic jump compared to the horizontal case. Early foundational work by (Bakhmeteff & Matzke, 1938) and (Kindsvater, 1944) first explored the hydraulic jump in sloping channels. (Argyropoulos, 1962) provided a generalized solution, while (Mikhalev & Hoang Ty An, 1976) and (Rajaratnam & Murahari, 1974) examined the kinematics and flow characteristics specific to sloping aprons. Subsequent research has expanded upon these findings, investigating the sequent depth ratio in sloped channels (Beirami & Chamani, 2006) and studying jumps in various non-rectangular sloping cross-sections, such as triangular channels (Debabeche et al., 2009) and U-shaped channels (Cherhabil & Debabeche, 2016). Further detailed studies have involved theoretical and experimental analyses in prismatic channels with variable slopes (Kateb, S., 2014) and sloped rectangular channels (Ghomri et al., 2018); (Nouacer, B., 2023). Methods have also been developed to accurately locate the toe of the jump in inclined channels (Guan-Yong Luo a et al., 2021). The influence of channel slope on jump properties, particularly in the context of environmental sustainability and design, has been highlighted in recent predictive and experimental evaluations (Gupta & Dwivedi, 2024; Gupta, S. K. & Dwivedi, V. K., 2025; Sanjeev Kumar Gupta & Vijay Kumar Dwivedi, 2023).

A second critical parameter affecting jump characteristics is bed roughness. Increased roughness enhances turbulence and boundary shear stress, leading to greater energy dissipation and changes in sequent depth relationships (Chanson, 2015). Early experimental work confirmed that roughness influences hydraulic jump properties (Hughes & Flack, 1984), particularly on corrugated beds (Ead & Rajaratnam, 2002). Recent investigations have explored the classification of jumps over rough beds (Mahtabi et al., 2020) and the combined effect of slope and roughness on open channel flow behavior (Laishram et al., 2021). The impact of roughness has been studied in sudden expansion channels (Daneshfaraz et al., 2021), in stilling basins with negative steps (Sayyadi et al., 2022), and over adverse slopes controlled by roughness elements (Parsamehr et al., 2022). Furthermore, studies focusing on D-jumps (Palermo & Pagliara, 2017) and semi-theoretical studies on conjugate depth ratio (khechiba et al., 2025) have confirmed the complex interaction between bed shear and jump parameters.

A third factor of engineering relevance is channel geometry, particularly the composite (or compound) cross-section, which models flow systems like rivers and flood-protection channels. The presence of major and minor beds introduces complexities in flow distribution and momentum transfer, which have been documented in studies concerning flow dynamics through contractions (Hachemi Rachedi, L., 2005). Research dedicated specifically to the hydraulic jump in composite channels has examined the jump structure and characteristics (Khattaoui, M. & Achour, 2012); (BENABDESSELAM, 2020); (Riguet. F et al., 2020), including numerical simulations (Boudjelal et al., 2024) and experimental analysis of the sequent depth ratio (Djamaa et al., 2022).

The individual effects of roughness within composite channels, such as a rough minor bed, have been previously investigated (Ghomri et al., 2023); (Lacheheb, S., 2023) studied a jump controlled by a weir in a composite channel with a rough major bed. (Hery et al., 2025) presented an experimental study of the hydraulic jump behavior in inclined composite channels with a rough secondary bottom. Despite this extensive body of literature covering the effects of slope, roughness, and composite geometry in isolation or in pairs, a significant gap remains regarding the comprehensive understanding of the hydraulic jump when all three factors -channel inclination, composite rectangular section, and roughness applied specifically to the major bed- are present simultaneously. The specific configuration of a rough major bed in an inclined rectangular composite channel introduces compounded shear forces and flow patterns that deviate significantly from idealized smooth, horizontal, or simple-section geometries, thereby challenging current predictive models.

Unlike previous studies that examined the effects of slope, roughness, or composite geometry separately, the present work provides the first integrated experimental assessment of hydraulic jump roller length in a sloped rectangular composite channel with roughness applied specifically to the major bed.

The principal scientific contribution of this study lies in the derivation of a unified global predictive model for the relative roller length (L_r/h_1), simultaneously incorporating upstream Froude number, relative roughness ratio ($\varepsilon/(B-b)$), and longitudinal slope ($\tan\alpha$).

This integrated formulation fills a significant gap in hydraulic jump modeling under realistic engineering conditions where slope and roughness coexist within compound cross-sections.

2. Experimental Setup and Methodology

Experimental runs investigating hydraulic jumps over a rough major bed in a compound rectangular channel were conducted at the Laboratory for the Exploitation and Development of Natural Resources in Arid Zones (EVRNZA), University of Ouargla. The experimental apparatus, illustrated in Figure 3, consists of a 10-m-long, 0.5-m-deep free-surface flume with transparent Plexiglas sidewalls to facilitate flow observation. The test section spans a length of 4 m and features a compound rectangular cross-section with a total height (h) of 15.5 cm, comprising a minor bed height (b) of 14.4 cm and a major bed width (B) of 25 cm. While the channel bed is structurally planar, the flume is equipped with an upstream mechanical tilting system. This screw-jack mechanism allows for precise adjustment of the longitudinal slope, covering values of $\tan(\alpha) = 0, 0.005, 0.01$, and 0.015. Water is supplied from a closed metal basin via a 150-mm diameter pipe, where an adjustable inlet sluice gate regulates the entry conditions to generate a supercritical flow regime. The initial flow depth is controlled to correspond with the initial ledge height (h_1).

A constant discharge of 55.55 L/s is maintained by an axial pump and regulated via a control valve. At the downstream end, flow rates are quantified using a standard rectangular weir. Consequently, the specific discharge is derived by substituting the measured head (h) into the discharge equation established by (Hachemi Rachedi, L., 2005), expressed as:

$$Q = 0{,}3794B\sqrt{2g}\beta(1 + 0{,}16496\beta^{2{,}0716})^{3/2}\, h_{dev}^{3/2} \tag{1}$$

The experimental matrix encompassed four initial inflow depths (h_1= 2.5, 3, 3.5, and 4 cm) and four bed roughness coefficients (ε (mm) = 6, 8, 10, and 12 mm), allowing for a comprehensive analysis of the hydraulic jump characteristics under varying boundary conditions.

2.1. Definition of the relative roller length (Lr/h_1)

The toe of the hydraulic jump was identified visually as the location of abrupt water surface rise and confirmed through depth discontinuity measurements. The roller termination point was determined where surface recirculation visibly disappeared and the flow transitioned to quasi-uniform subcritical conditions. Each experimental condition was repeated three times, and averaged values were reported to reduce random measurement errors.

The measurement uncertainty was estimated based on instrument precision. while longitudinal distances were measured with an accuracy of ±2 mm. Propagated uncertainty in Lr/h_1 was estimated to be below 3%.

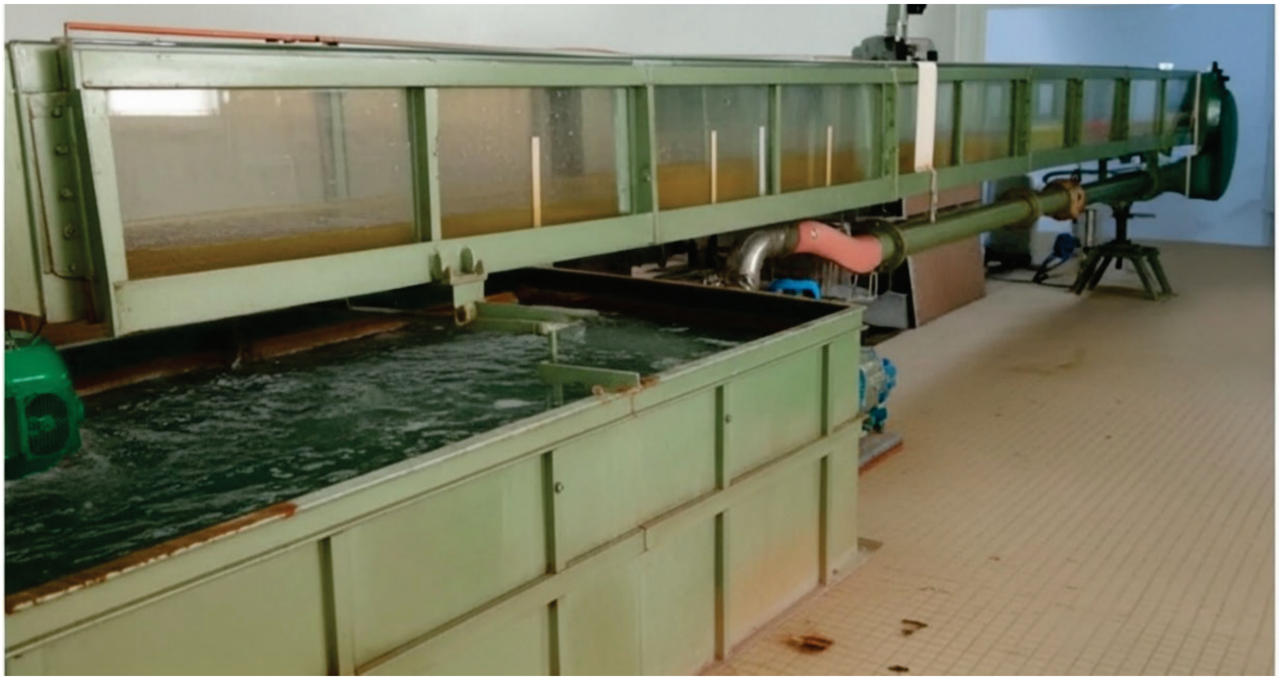

Figure 1. Overview of the test channel

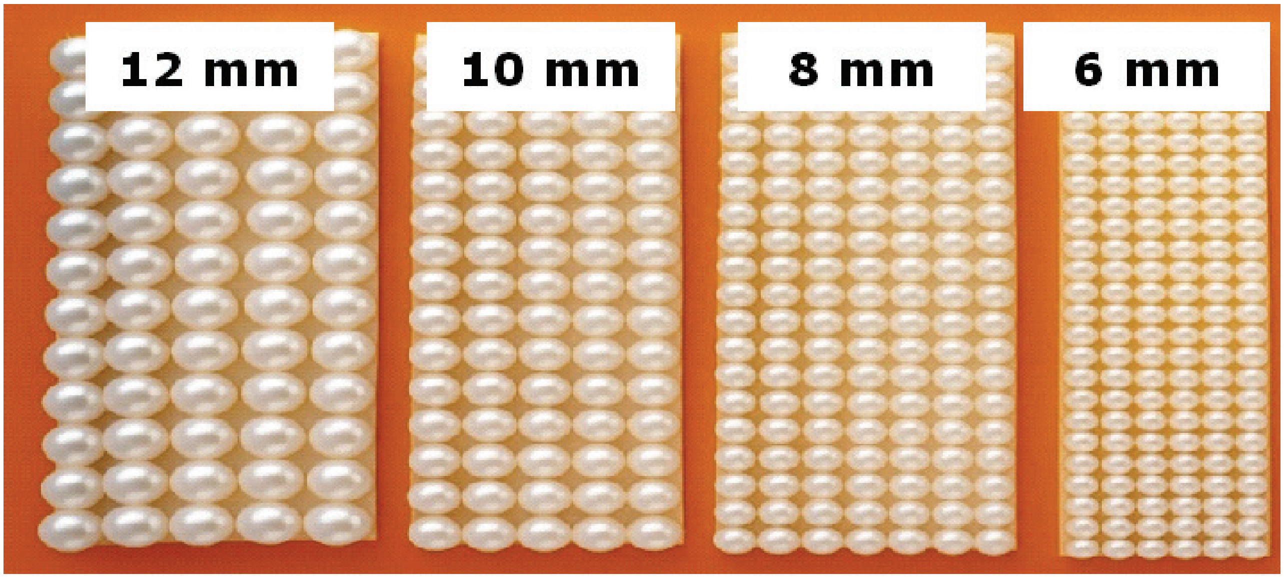

Figure 2. Artificial roughness mats with varying spherical bead diameters (6, 8, 10, and 12 mm).

Figure 3. Experimental setup of the compound channel with roughened floodplains (major bed) and a smooth main channel.

3. Results and discussion

3.1. Variation of the relative roller length of the jump (Lr/h_1) as a function of the Froude number F_1 and the channel's inclination angle (α)

Figures 4–7 present the experimental data concerning the relative roller length (Lr/h_1) plotted against the Froude number (F_1). The analysis was conducted under varying hydraulic and geometric conditions, comprising four inclination angles defined by tang (α) = 0; 0,005; 0,01; 0.015 and five absolute roughness coefficients ε (mm) taking values of 0, 6, 8, 10, and 12 mm.

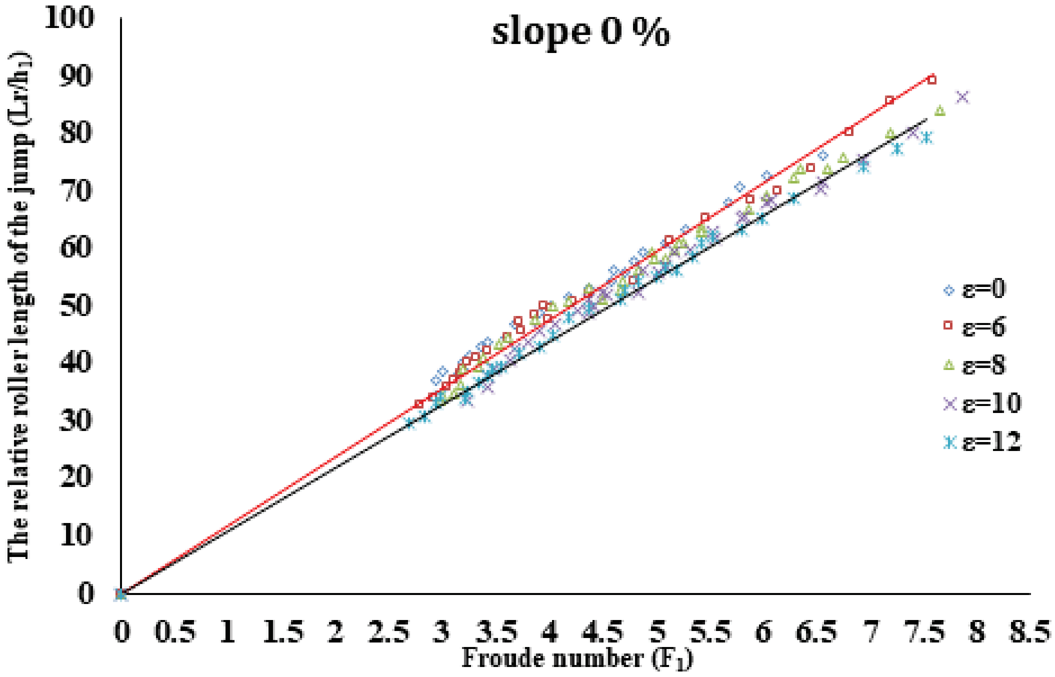

Figure 4. Variation in the relative roller length with the Froude number for slope = 0% at various roughness.

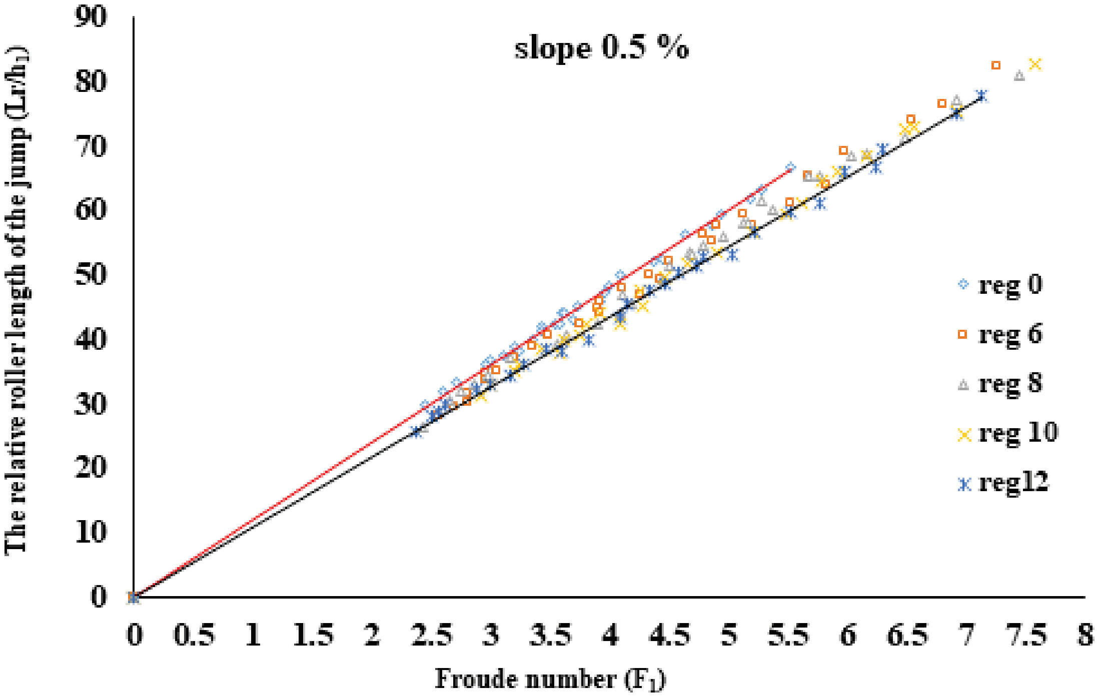

Figure 5. Variation in the relative roller length with the Froude number for slope =0.5% at various roughness.

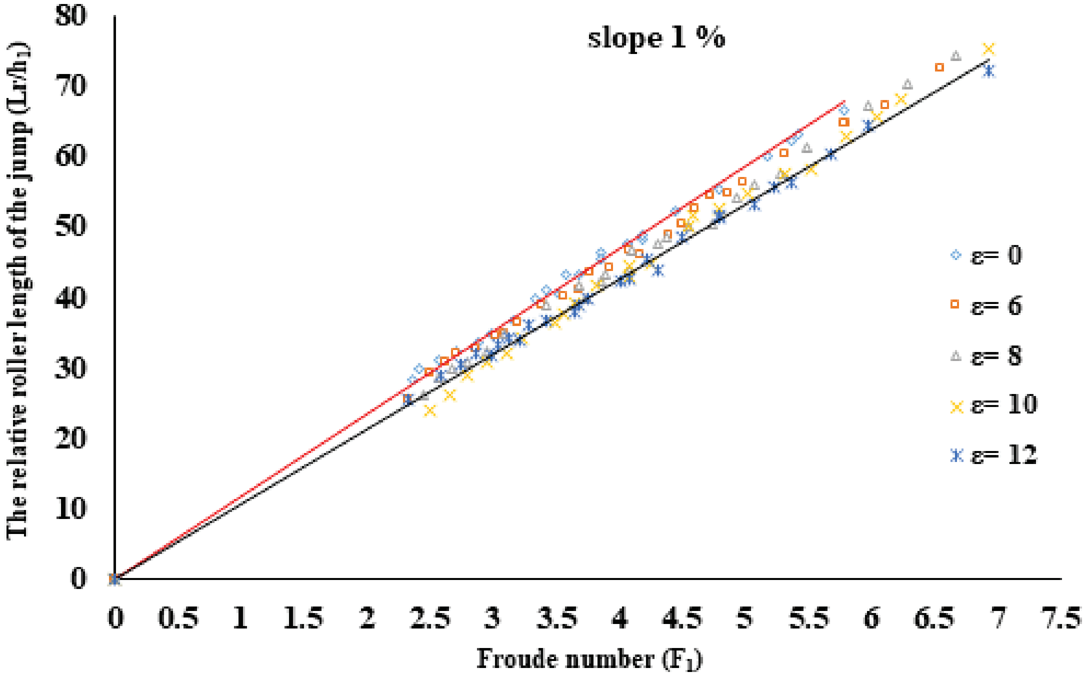

Figure 6. Variation in the relative roller length with the Froude number for slope =1% at various roughness.

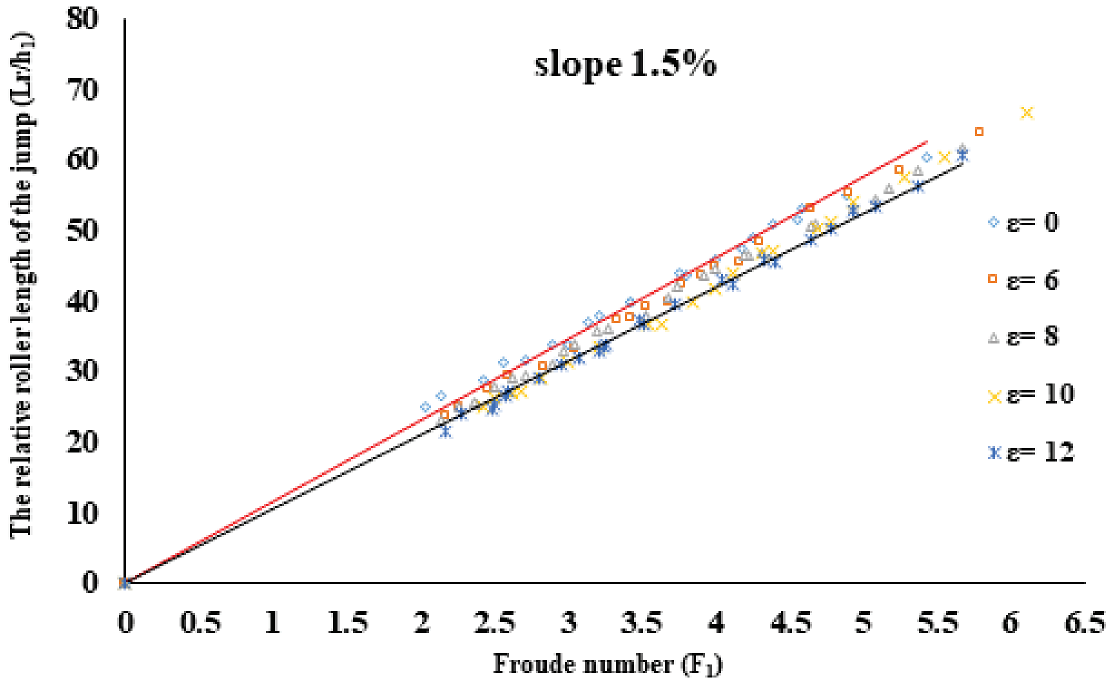

Figure 7. Variation in the relative roller length with the Froude number for slope =1.5% at various roughness.

The experimental data collected for hydraulic jumps over a rough floodplain are categorized into five distinct series, each corresponding to a constant absolute roughness ε (mm). Regression analysis reveals that for each roughness configuration, the data points align with a linear function of the form $Lr/h_1 = a \cdot F_1$. This linearity underscores a direct correlation between the roller geometry and the upstream flow intensity.

Crucially, the analysis highlights two opposing trends: while the relative roller length (Lr/h_1) increases monotonically with the Froude number (F_1), it exhibits an inverse relationship with bed roughness. Specifically, an increase in the relative roughness ratio (ε/(B-b)) leads to a significant reduction in the (Lr/h_1) ratio. This attenuation in roller length can be attributed to the enhanced shear stress and turbulence induced by the roughness elements, which accelerate energy dissipation. The correlation coefficients and specific parameters derived from this analysis are summarized in Tables 1–4.

Table 1. Values of the parameters a_1

Slope 0 %		
ε/(B-b)	**a_1**	**R^2**
0.000	12.137	0.9994
0.057	11.669	0.9994
0.075	11.319	0.999
0.094	11.141	0.9993
0.113	10.945	0.9995

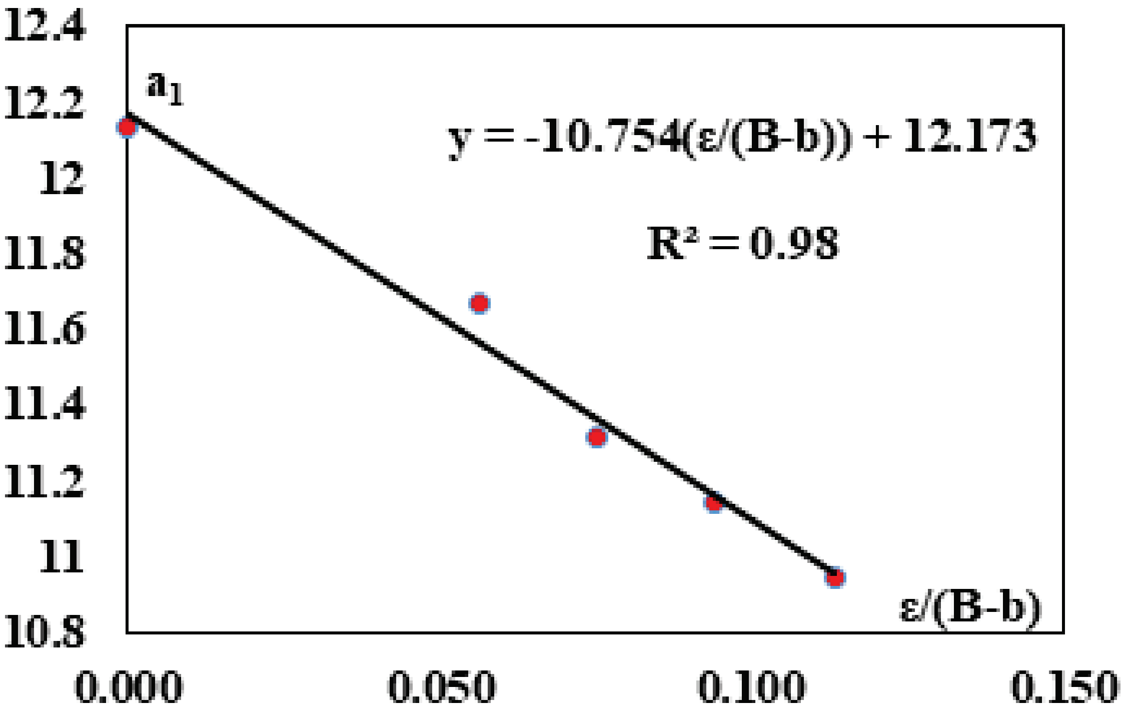

Figure 8. Correlation of the a_1 factor with the ε/(B-b) ratio

The analysis of the experimental measurements of the pairs of values (ε/(B-b), a_1) leads to the elaboration of the following explicit approach: $a_1 = -10.754(\varepsilon/(B-b)) + 12.173$.

Table 2. Values of the parameters a_2

Slope 0.5 %		
ε/(B-b)	**a_2**	**R^2**
0.000	12.005	0.9997
0.057	11.367	0.9994
0.075	11.219	0.9995
0.094	10.987	0.9997
0.113	10.866	0.9995

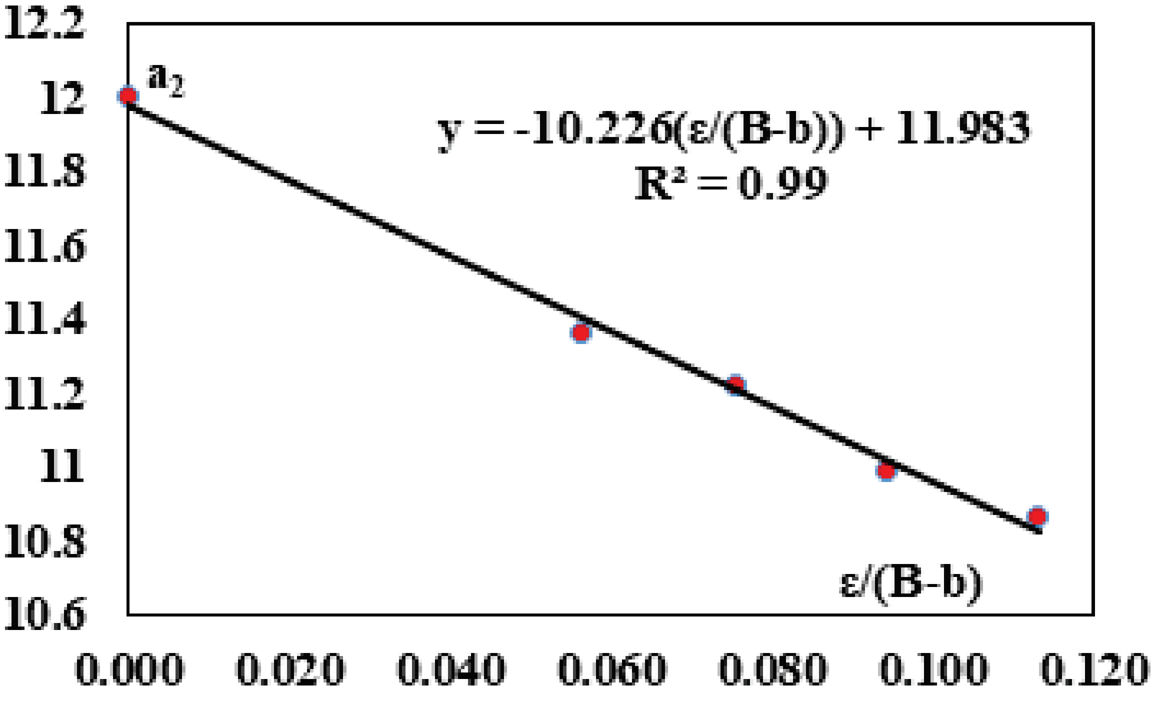

Figure 9. Correlation of the a_2 factor with the ε/(B-b) ratio

The analysis of the experimental measurements of the pairs of values (ε/(B-b), a_2) leads to the elaboration of the following explicit approach: $a_2 = -10.226(\varepsilon/(B-b)) + 11.983$.

Table 3. Values of the parameters a_3

Slope 1 %		
ε/(B-b)	**a_3**	**R^2**
0.000	11.710	0.9997
0.057	11.267	0.9996
0.075	11.055	0.9998
0.094	10.755	0.9994
0.113	10.634	0.9996

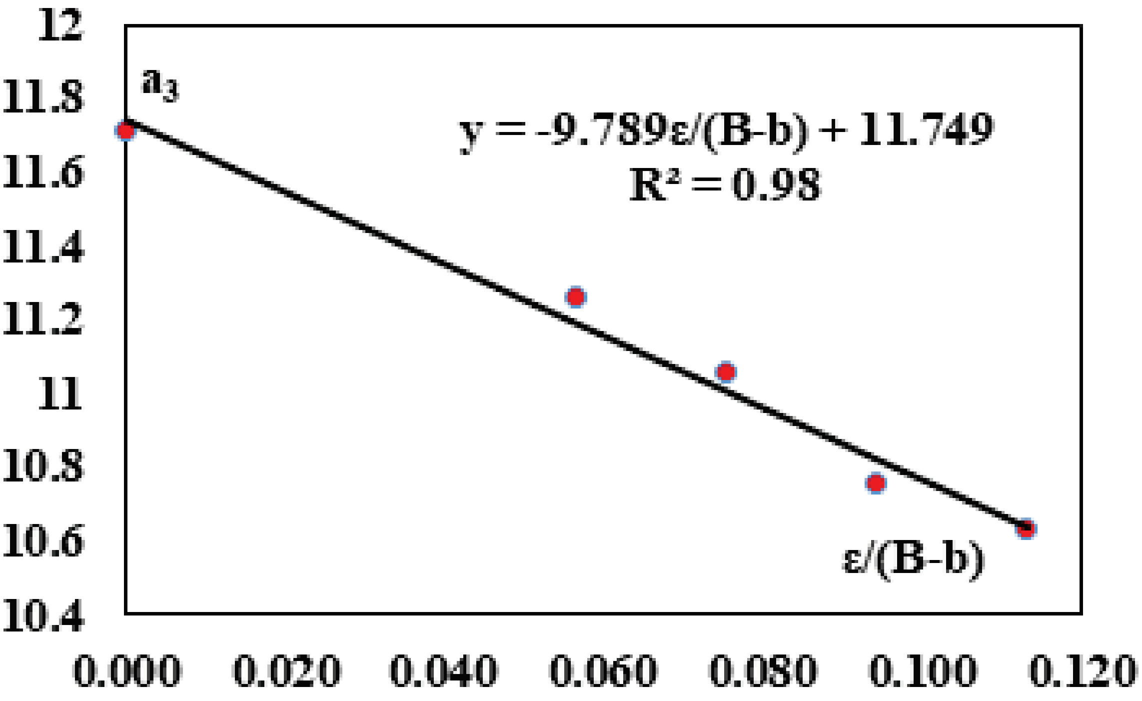

Figure 10. Correlation of the a_3 factor with the ε/(B-b) ratio

The analysis of the experimental measurements of the pairs of values (ε/(B-b), a_3) leads to the elaboration of the following explicit approach: $a_3 = -9.789(\varepsilon/(B-b)) + 11.749$'.

Table 4. Values of the parameters a_4

Slope 1.5 %		
ε/(B-b)	**a_4**	**R^2**
0.000	11.525	0.9995
0.057	11.122	0.9998
0.075	10.917	0.9998
0.094	10.667	0.9994
0.113	10.487	0.9997

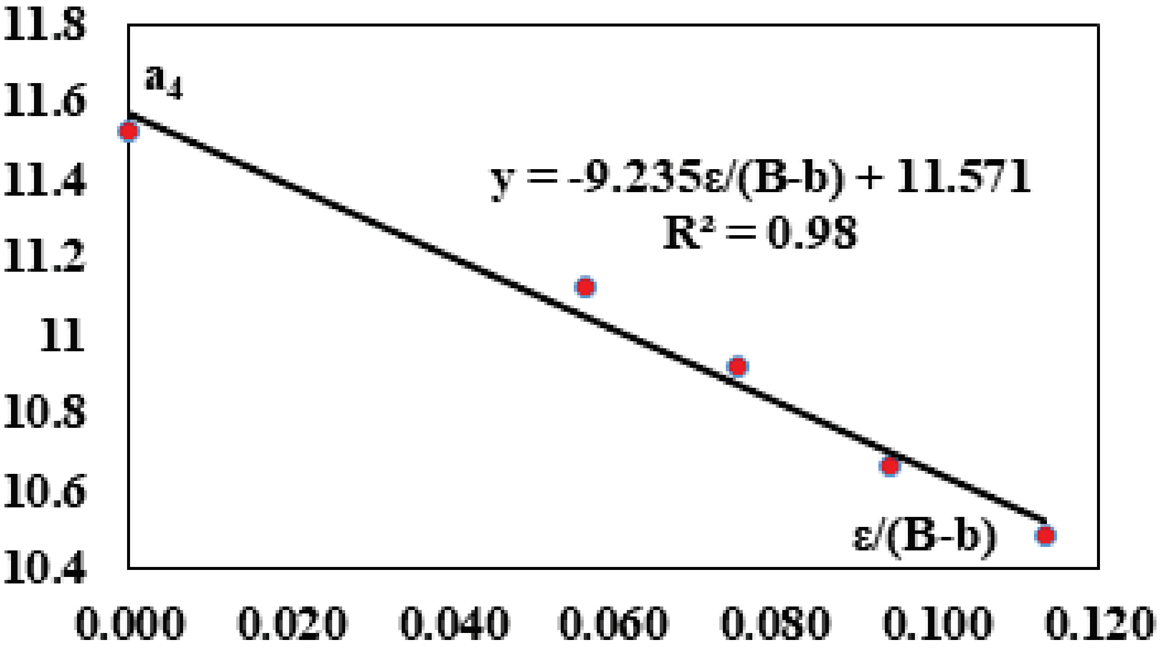

Figure 11. Correlation of the a_4 factor with the ε/(B-b) ratio

The analysis of the experimental measurements of the pairs of values (ε/(B-b), a_4) leads to the elaboration of the following explicit approach: $a_4 = -9.2345(\varepsilon/(B-b)) + 11.571$.

Replacing the parameters a with their respective expressions in the relation $Lr/h_1 = a\ F_1$, we obtain the following:

$$\mathrm{Lr/h_1} = (-10.754\varepsilon/(\mathrm{B}-\mathrm{b}) + 12.173)\mathrm{F_1}\ ;\ \tan\alpha = 0\% \tag{2}$$

$$\mathrm{Lr/h_1} = (-10.226\varepsilon/(\mathrm{B}-\mathrm{b}) + 11.983)\mathrm{F_1}\ ;\ \tan\alpha = 0.5\% \tag{3}$$

$$\mathrm{Lr/h_1} = (-9.789\varepsilon/(\mathrm{B}-\mathrm{b}) + 11.749)\mathrm{F_1}\ ;\ \tan\alpha = 1\% \tag{4}$$

$$\mathrm{Lr/h_1} = (-9.235\varepsilon/(\mathrm{B}-\mathrm{b}) + 11.571)\mathrm{F_1}\ ;\ \tan\alpha = 1.5\% \tag{5}$$

The table below shows the values of a' and b' of the roughness coefficient with the angle of inclination of the channel.

Table 5. Values a' and b' of the correlated roughness coefficient to the angle of inclination of the canal.

ε /(B-b) (mm)	**0 ; 6 ; 8 ; 10 ; 12**		
tan (α)	**a`**	**b`**	**R^2**
0 %	-10.754	12.173	0.98
0.5 %	-10.226	11.983	0.99
1 %	-9.789	11.749	0.98
1.5 %	-9.235	11.571	0.98

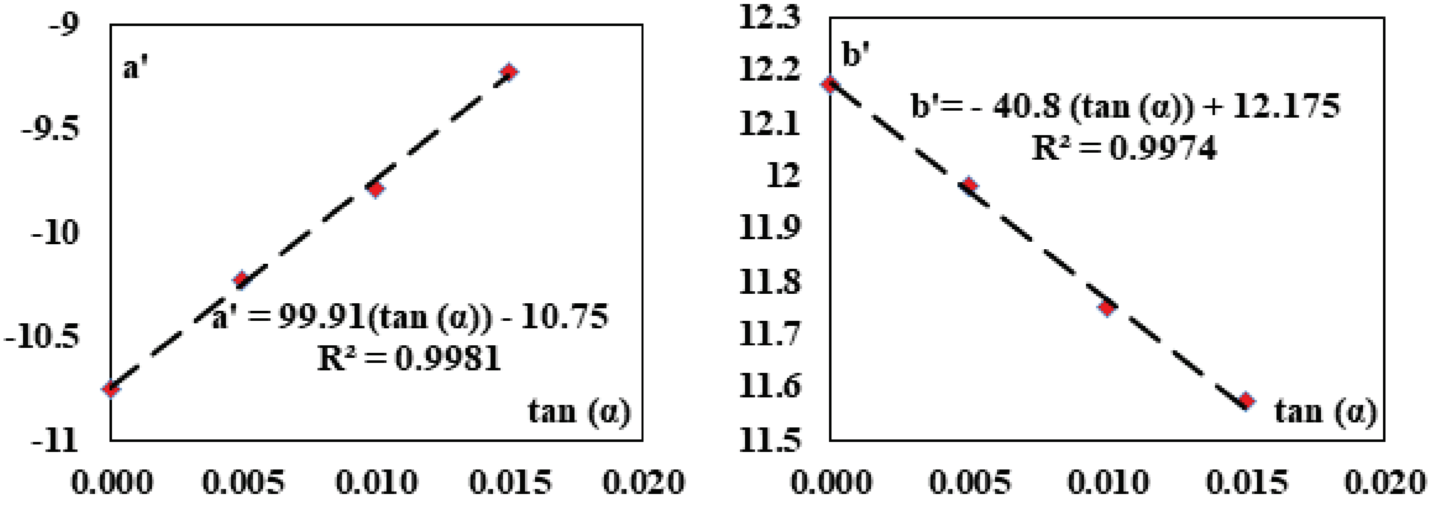

Figure 12. Variation of the factors a' and b' as a function of tan (α)

$$\text{a}` = 99.91\tan(\alpha) - 10.75 \quad R^2 = 0.99 \tag{6}$$

$$\text{b}` = -40.8\tan(\alpha) + 12.175 \quad R^2 = 0.99 \tag{7}$$

It would be convenient to reformulate the four relationships into an overall equation in the following form:

$$\text{Lr}/\text{h}_1 = [(99.91(\tan(\alpha)) - 10.75)(\varepsilon/(\text{B}-\text{b})) - 40.8(\tan(\alpha)) + 12.175]\text{F}_1 \tag{8}$$

$$2.03 \leq F_1 \leq 7.87 \quad 0{,}000 \leq \frac{\varepsilon}{B-b} \leq 0.113 \quad 0 \leq tan\ (\alpha) \leq 0.015$$

Classical hydraulic jump formulations developed for horizontal smooth beds (Rajaratnam, 1967; Hager & Bremen, 1989) generally predict longer roller lengths compared to the present findings. The discrepancy arises because increased bed roughness enhances boundary shear stress and turbulence production, accelerating momentum exchange and shortening the recirculation zone. Furthermore, the presence of channel slope introduces a gravitational component along the flow direction, modifying pressure distribution and energy balance. The combined action of these mechanisms explains the systematic reduction in Lr/h_1 observed in the present experiments compared to traditional smooth-bed correlations.

The validity of the comparison between the overall dimensionless approach $Lr/h_{1relation}$ and the experimental Lr/h_{1exp} values is illustrated in Figure 13. The satisfactory alignment of the data on the first bisector confirms the relevance of this functional modeling.

In addition, the relative errors observed, mostly less than 5%, reflect a satisfactory precision of the calculations while facilitating the use of the results. The particularly high correlation coefficient ($R^2 = 0.9985$) reflects the structured dimensional formulation adopted and the controlled laboratory conditions under which experiments were conducted. However, it is acknowledged that further validation under independent experimental or field-scale conditions would strengthen the general applicability of the model. The accuracy of the model is also evidenced by a small mean absolute error (MAE = 0.83). Finally, its consistency and reliability are confirmed by the small average relative error (RE = 0.019) as well as by the low dispersion of the latter, characterized by its standard deviation (σRE = 0.065).

Table 6. Comparison between the proposed model and selected hydraulic jump length correlations

Reference	Channel Type	Slope Considered	Roughness Considered	Main Predictors	Applicability	Observed Limitation
Rajaratnam (1967)	Rectangular, smooth	No	No	F_1	Horizontal channels	Overestimates Lr under rough or sloped conditions
Hager & Bremen (1989)	Rectangular	No	No	F_1, y_2/y_1	Smooth beds only	Does not include slope or roughness effects
Ead & Rajaratnam (2002)	Rough beds	No	Yes	F_1, roughness	Horizontal only	No slope effect
Palermo & Pagliara (2017)	Rough sloping channels	Yes	Yes	F_1, slope, roughness	Simple cross-sections	Not applied to composite geometry
Gupta & Dwivedi (2024)	Sloping rough surfaces	Yes	Yes	F_1, slope	Rectangular uniform	Does not consider compound sections
Present Study	Sloped composite rectangular	Yes	Yes (major bed)	F_1, tan α, $\varepsilon/(B-b)$	Composite channel with rough major bed	Valid within tested parameter range

The comparison clearly demonstrates that most classical formulations were developed either for smooth horizontal beds or for simple rough rectangular geometries. None of the reviewed models simultaneously integrate the combined effects of channel slope, composite geometry, and localized major-bed roughness.

The present formulation extends existing approaches by incorporating these interacting parameters within a unified predictive framework. This explains the systematic deviation observed when classical correlations are applied to the current experimental configuration, particularly the overestimation of roller length under enhanced turbulence conditions.

Consequently, the proposed model provides improved predictive capability under realistic engineering conditions where slope and roughness coexist in compound channels.

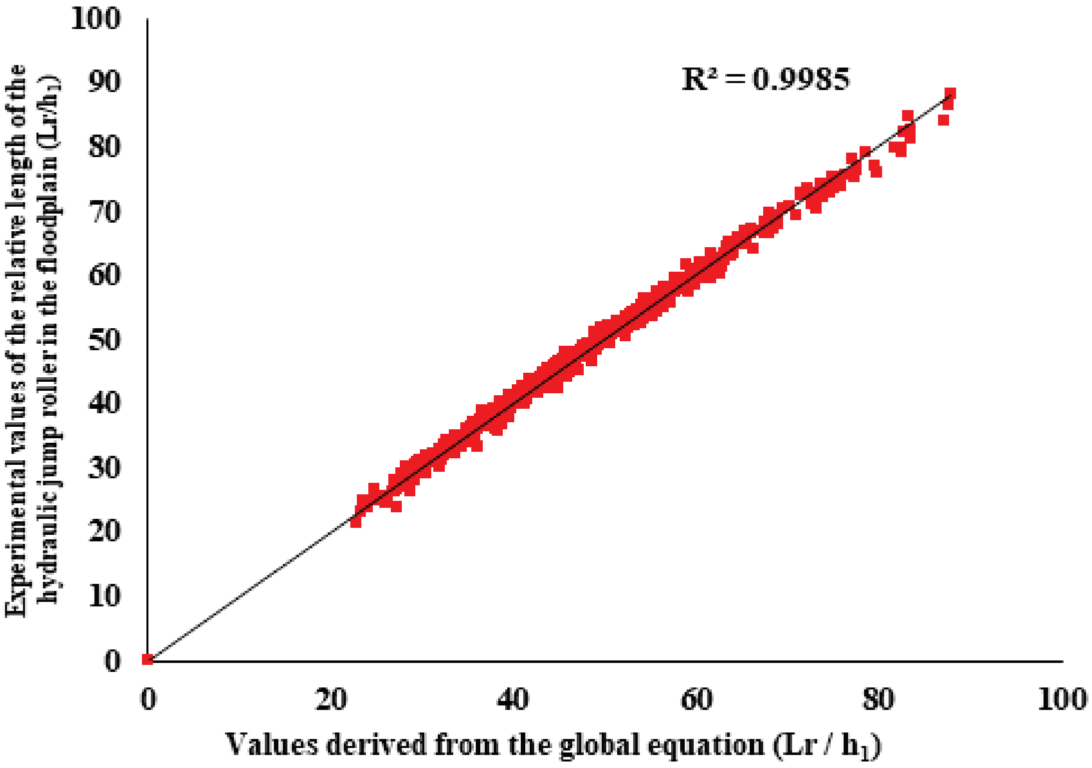

Figure 13. Comparison between experimental Lr/h_1 and the global approach f(tan (α),ε/(B-b), F_1)

4. Conclusion

This study provides a comprehensive experimental evaluation of hydraulic jump roller length in sloped composite rectangular channels with major-bed roughness. The integration of slope, roughness, and compound geometry within a single predictive framework represents a significant advancement in hydraulic jump modeling under realistic engineering configurations. Key findings indicate that the relative roller length (L_r/h_1) exhibits a direct correlation with the upstream Froude number, confirming the expected behavior of the hydraulic jump in varying flow intensities. Additionally, bed roughness significantly influences jump characteristics, as increasing roughness reduces the relative roller length. This result can be attributed to enhanced shear stress and turbulence from the rough bed, which accelerates energy dissipation and alters flow patterns. Importantly, an inverse relationship was observed between bed roughness and jump length, further underscoring the role of turbulence in controlling jump dynamics.

The study also developed a comprehensive predictive model for the hydraulic jump characteristics, incorporating channel slope, roughness, and Froude number. This model demonstrated high precision in predicting jump length across the experimental conditions, with a correlation coefficient of 0.9985 and minimal mean absolute error (MAE = 0.83). The proposed equation, which accounts for the combined effects of slope and roughness on the jump dynamics, provides a valuable tool for engineering applications, particularly in the design of hydraulic structures such as spillways and sluice gates.

While the study provides significant insights, there are limitations that warrant further investigation. The experimental setup focused on specific roughness values and slope conditions, and future research could explore a broader range of geometries, including more complex composite channel designs and variable flow conditions. Additionally, the model could be further refined by incorporating dynamic flow behaviors under more diverse operational regimes, such as transient flow conditions or during extreme events like flooding.

Despite the robustness of the proposed model within the investigated parameter range, caution should be exercised when extrapolating beyond the tested slopes, roughness sizes, or Froude numbers. Future work should include field validation and extension toward transient and non-uniform inflow conditions.

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6. References

Alghwail, A. D. A. (2021). Investigation of the Perfect Hydraulic Jump in Horizontal Rectangular Channel. *Journal of Alasmarya University*, 6(5), 222–229. https://doi.org/10.59743/aujas.v6i5.1144

Argyropoulos, P. A. (1962). General Solution of the Hydraulic Jump in Sloping Channels. *Journal of the Hydraulics Division*, 88(4), 61–75. https://doi.org/10.1061/JYCEAJ.0000767

Bakhmeteff, B. A., & Matzke, A. E. (1938). The Hydraulic Jump in Sloped Channels. *Transactions of the American Society of Mechanical Engineers*, 60(2), 111–118. https://doi.org/10.1115/1.4020643

Beirami, M. K., & Chamani, M. R. (2006). Hydraulic Jumps in Sloping Channels: Sequent Depth Ratio. *Journal of Hydraulic Engineering*, 132(10), 1061–1068. https://doi.org/10.1061/(ASCE)0733-9429(2006)132:10(1061)

BENABDESSELAM, A. (2020). Approches theoriques et experimentales du ressaut hydraulique dans un profil de canal compose (doctoral thesis). Université Mohamed Khider – Biskra. http://thesis.univ-biskra.dz/id/eprint/4758

Boudjelal, S., Fourar, A., & Massouh, F. (2024). Numerical Simulation of Hydraulic Jump in a Compound Channel. Engineering. *Technology & Applied Science Research*, 14(5), 17393–17397. https://doi.org/10.48084/etasr.8235

Chanson, H. (2015). *Energy dissipation in hydraulic structures*. CRC Press/Taylor & Francis Group. https://doi.org/10.1201/b18441

Chen-Feng Li. (1995). Determining the location of hydraulic jump by lmodel test and hec-2 flow routing (Master of Science). College of Engineering and Technology, Ohio University. https://etd.ohiolink.edu/acprod/odb_etd/ws/send_file/send?accession=ohiou1179944723&disposition=inline

Cherhabil, S., & Debabeche, M. (2016). *Experimental Study of Sequent Depths Ratio of Hydraulic Jump in Sloped Trapezoidal Chanel*. https://doi.org/10.15142/T3610628160853

Daneshfaraz, R., MajediAsl, M., Mirzaee, R., & Tayfur, G. (2021). Hydraulic Jump in a Rough Sudden Symmetric Expansion Channel. *AUT Journal of Civil Engineering*, 5(2), 245–256. https://doi.org/10.22060/ajce.2020.18227.5667

Debabeche, M., Cherhabil, S., Hafnaoui, A., & Achour, B. (2009). Hydraulic jump in a sloped triangular channel. *Canadian Journal of Civil Engineering*, 36(4), 655–658. https://doi.org/10.1139/L08-136

Djamaa, W., Ghomri, A., & Al-Fawzy, A. (2022). Experimental study of the sequent depths ratio of the hydraulic jump in a rectangular compound channel with a rough minor bed. *LARHYSS Journal P*, 49, Article 49.

Ead, S. A., & Rajaratnam, N. (2002). Hydraulic Jumps on Corrugated Beds. *Journal of Hydraulic Engineering*, 128(7), 656–663. https://doi.org/10.1061/(ASCE)0733-9429(2002)128:7(656)

Ghomri, A., Kateb, S., & Riguet, F. (2018). Contribution a l'etude experimentale du ressaut hydraulique controle par seuil dans un canal rectangulaire a pente variable. http://dspace.univ-eloued.dz:80/xmlui/handle/123456789/1662

Ghomri, A., Lacheheb, S., & Djamaa, W. (2023). Effect of roughness on the characteristics of the hydraulic jump evolving into a rectangular channel of compound section. *Journal of Fundamental and Applied Sciences*, 13, 50. https://doi.org/10.4314/jfas.1300

Guan-Yong Luo a, , Hong Caoa, & , and Hong Pana. (2021). Method to locate the toe of a hydraulic jump on sloping channels. *KSCE Journal of Civil Engineering*, 25. https://doi.org/10.1007/s12205-020-0081-7

Gupta, S. K., & Dwivedi, V. K. (2024). Experimental investigation of hydraulic jump characteristics in sloping rough surfaces for sustainable development. *Engineering Research Express*, 6(2), 025103. https://doi.org/10.1088/2631-8695/ad3acf

Gupta, S. K. & Dwivedi, V. K. (2025). Evaluation of hydraulic jump characteristics in rough sloping surfaces for sustainable environment: A laboratory investigation. *Sigma Journal of Engineering and Natural Sciences*, 148–159. https://doi.org/10.14744/sigma.2025.00012

Hachemi Rachedi, L. (2005). Analyse d'un écoulement au travers d'une contraction latérale (magister en hydraulique). Université de Biskra. http://thesis.univ-biskra.dz/id/eprint/1102

Hafnaoui, M. A., & Debabeche, M. (2023). Displacement of a Hydraulic Jump in a Rectangular Channel: Experimental Study. *Iranian Journal of Science and Technology, Transactions of Civil Engineering*, 47(2), 1181–1188. https://doi.org/10.1007/s40996-022-00974-y

Hager, W. H., & Bretz, N. V. (1987). Discussion of "Simplified Design of Contractions in Supercritical Flow" by Terry W. Sturm (May, 1985, Vol. 111, No. 5). *Journal of Hydraulic Engineering*, 113(3), 422–424. https://doi.org/10.1061/(ASCE)0733-9429(1987)113:3(422)

Herri, I., Samir, K., Ali, G., Haroun, K., & Djamel, B. (2025). Experimental investigation of the sequent depth ratio in hydraulic jumps occurring in a sloped compound rectangular channel with a rough minor bed. *Water Practice & Technology*, 20(8), 1745–1762. https://doi.org/10.2166/wpt.2025.101

Hughes, W. C., & Flack, J. E. (1984). Hydraulic Jump Properties Over a Rough Bed. *Journal of Hydraulic Engineering*, 110(12), 1755–1771. https://doi.org/10.1061/(ASCE)0733-9429(1984)110:12(1755)

Kateb, S. (2014). Etude theorique et experimentale de quelques types de ressauts hydrauliques dans un canal trapezoïdal (Thèse de doctorat en sciences). Université Mohamed Khider, Biskra, Algérie. http://thesis.univ-biskra.dz/id/eprint/63

Khattaoui, M. & Achour. (2012). Ressaut hydraulique dans un canal rectangulaire compose. *Revue Scientifique et Technique LJEE*, No. 20, 44–51.

khechiba, haroun, Ghomri, A., Besser, D., & Herri, I. (2025). A Semi-Theoretical Study on the Effect of Rough Beds on the Conjugate Depth Ratio in Hydraulic Jumps. *Engineering, Technology & Applied Science Research*, 15(3), 23082–23087. https://doi.org/10.48084/etasr.10754

Kindsvater, C. E. (1944). The Hydraulic Jump in Sloping Channels. *Transactions of the American Society of Civil Engineers*, 109(1), 1107–1120. https://doi.org/10.1061/TACEAT.0005733

Lacheheb, S., (2023). Contribution à l’étude du ressaut hydraulique contrôlé par seuil à paroi mince évoluant dans un canal rectangulaire de section composée avec lit majeur rugueux (Thèse de doctorat). UNIVERSITE KASDI MERBAH OUARGLA. https://dspace.univ-ouargla.dz/jspui/handle/123456789/33184

Laishram, K., Albino Kumar, P., & Tamphasana Devi, T. (2021). Effect of channel slope and roughness on hydraulic jump in open channel flow. *IOP Conference Series: Earth and Environmental Science*, 958, 012014. https://doi.org/10.1088/1755-1315/958/1/012014

Mahtabi, G., Chaplot, B., Azamathulla, H. M., & Pal, M. (2020). Classification of Hydraulic Jump in Rough Beds. *Water*, 12(8), Article 8. https://doi.org/10.3390/w12082249

Mikhalev, M. A. & Hoang Ty An. (1976). Kinematic characteristics of a hydraulic jump on a sloping apron. *Hydrotechnical Construction*, 10(7), 686–690. https://doi.org/10.1007/BF02381815

Nandi, B., Das, S., & Mazumdar, A. (2020). Experimental Analysis and Numerical Simulation of Hydraulic Jump. *IOP Conference Series: Earth and Environmental Science*, 505(1), 012024. https://doi.org/10.1088/1755-1315/505/1/012024

Nouacer, B. (2023). Contribution a l'etude theorique et experimentale de ressaut hydraulique evoluant dans un canal rectangulaire incline a fond rugueux (Thèse de doctorat). UNIVERSITE KASDI MERBAH OUARGLA. https://dspace.univ-ouargla.dz/jspui/handle/123456789/33312

Palermo, M., & Pagliara, S. (2017). D-jump in rough sloping channels at low Froude numbers. *Journal of Hydro-Environment Research*, 14, 150–156. https://doi.org/10.1016/j.jher.2016.10.002

Parsamehr, P., Kuriqi, A., Farsadizadeh, D., Dalir, A. H., Daneshfaraz, R., & Ferreira, R. M. L. (2022). Hydraulic jump over an adverse slope controlled by different roughness elements. *Water Resources Management*, 36(14), 5729–5749. https://doi.org/10.1007/s11269-022-03330-x

Rajaratnam, N. (1965). Hydraulic Jump in horizontal conduit. *Water Power*, 17, 80–83.

Rajaratnam, N. (1967). Hydraulic Jumps. In *Advances in Hydroscience* (Vol. 4, pp. 197–280). Elsevier. https://doi.org/10.1016/B978-1-4831-9935-1.50011-2

Rajaratnam, N., & Murahari, V. (1974). Flow Characteristics of Sloping Channel Jumps. *Journal of the Hydraulics Division*, 100(6), 731–740. https://doi.org/10.1061/JYCEAJ.0003975

Riguet. F, Debabeche . M, & A. Ghomri. (2020). Experimental study of the sequent depth ratio of the hydraulic jump in a straight compound rectangular channel. *Journal of Fundamental and Applied Sciences*, 12(IS), 56–65. https://www.jfas.info/index.php/JFAS/article/view/633

Sanjeev Kumar Gupta & Vijay Kumar Dwivedi. (2023). Prediction of Depth Ratio, Jump Length and Energy Loss in Sloped Channel Hydraulic Jump for Environmental Sustainability. *Evergreen*, 10(2), 942–952. https://doi.org/10.5109/6792889

Sayyadi, K., Heidarpour, M., & Ghadampour, Z. (2022). Effect of Bed Roughness and Negative Step on Characteristics of Hydraulic Jump in Rectangular Stilling Basin. *Shock and Vibration*, 2022(1), 1722065. https://doi.org/10.1155/2022/1722065

Simsek, O., Akoz, M. S., & Oksal, N. G. S. (2023). Experimental analysis of hydraulic jump at high froude numbers. *Sādhanā*, 48(2), 47. https://doi.org/10.1007/s12046-023-02081-8